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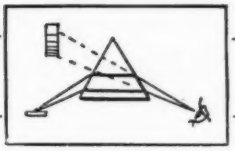

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For further information concerning the Optical Society of America address the Secretary, Dr. L. B. Tuckerman, National Bureau of Standards, Washington, D. C.

Journal of the
OPTICAL SOCIETY *of* AMERICA

VOLUME 27 NOVEMBER, 1937 NUMBER 11

A Precision, All-Purpose Microcamera[1]

L. C. GRATON AND E. B. DANE, JR.
Harvard University, Cambridge, Massachusetts
(Received June 11, 1937)

This microscope, intended primarily for photography with transmitted or reflected light, and particularly at high powers, departs widely from conventional design, especially in massive structure, appropriate materials, and high precision. Its most novel part is the motor-driven fine focusing mechanism which is 100 times slower than on standard instruments; 1 scale division = 0.01 micron or 100A. With this precise control of focus and with material for investigation believed superior to any hitherto employed for testing lens quality, results are secured and illustrated which indicate that true depth of focus is substantially less and resolution substantially greater than current theory would permit. Thus, change of focus of 1000A can be plainly recognized; and a colored structure 1070A wide is revealed in a photograph at 3175 diameters. It is inquired whether existing microscopical theory may not need review. Since present lenses apparently surpass their theoretical limits, a plea is made for better lenses still.

OUTLINE

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The microscope
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INTRODUCTION

MANY years of contact with microphotography as applied to the fields of petrography and ore deposition in the Laboratory of Mining Geology at Harvard had emphasized the advantages and satisfactions of this method of study and record. But simultaneously there had been brought clearly and aggravatingly into prominence certain limitations and shortcomings of existing equipment for the purposes needed in our laboratory. Long accumulated ideas regarding a new instrument had reached such tangible form that by the middle of 1931 it was decided to proceed with their conversion into reality. The instrument was sufficiently fabricated to permit

[1] Presented by invitation as part of a symposium on the Microscope and Microscopy, Twentieth Annual Meeting, Philadelphia, Oct., 1935.

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exhibition of its essential performance at the meetings of the A. A. A. S. and other scientific societies in Cambridge at the end of 1932; but many minor changes and refinements resulting from experience in actual use have been progressively incorporated. Indeed, it is our hope that the instrument is so adaptable that it will never be "completed."

The instrument was built by the Mann Instrument Company (Mico Instrument Company, successors) of Cambridge. We are under the greatest obligation to Mr. D. W. Mann, founder of that firm, for constant counsel and invaluable advice flowing from his rich experience in instrument making of the highest grade; and also for the resourcefulness of Mr. K. W. Harper, the firm's designing engineer, and of Mr. B. W. St. Clair, president; as well as to the several expert machinists who conscientiously and with enthusiasm brought the instrument into actual being. We have received help in the application of optical theory to our problem from Dr. Martin Grabau and Dr. H. M. O'Bryan, and advice with respect to electrical matters from Mr. M. W. Carley, all now or formerly with the department of physics at Harvard. To Drs. L. R. Van Wert and A. B. Greninger of the department of metallurgy our appreciation is expressed for discussions of microphotographic technique. We gratefully record our deep indebtedness to Dr. F. F. Lucas of the Bell Telephone Laboratories, New York, for innumerable suggestions of value from his expert experience. His own wonderful metallographic instrument[2] was received from Germany after the essentials of our design had been carried too far into execution to permit vital modification, but we have profited with respect to many details from the opportunity he generously gave for careful inspection of his equipment. Our thanks are due, further, to Dr. J. R. Vilella, now of the United States Steel Research Laboratories, Kearney, New Jersey, for helpful suggestions regarding instrumental performance that has contributed to the well-known excellence of his microphotography. We express grateful appreciation to Dr. L. V. Foster of Bausch and Lomb, Dr. W. G. Marquette, until recently associated with the New York branch of Zeiss, and Dr. H. W. Zieler of Leitz in New York, for their technical advice and their aid in securing the cooperation of their respective firms in furnishing us with more or less special optical parts.

The cost of the instrument was met by grants from the Milton, the Shaler, and the Secondary Enrichment funds at Harvard, supplemented by private contribution. We record our gratitude for this generous support.

A substantially identical microcamera has just now (1937) been finished for Dr. M. H. Haycock of the Department of Mines of the Dominion of Canada, at Ottawa. The building of the microscope proper was entrusted to the Mico Instrument Company, with whom the present writers have gladly cooperated to insure incorporation in the new instrument of all the modifications which four years of experience with our instrument have shown desirable. Later footnotes mention the principal individual changes embodied in the Ottawa instrument.

Governing Principles of the Design

Purpose and scope

It is to be emphasized at the outset that this is in no sense a stock instrument; in development of the design, therefore, it was never necessary to subordinate precision, efficiency or wide range of applicability to such considerations as cost, convenience in manufacture, or marketability.

The problems of our laboratory call for a wide variety of attack instead of a few routine procedures. We require an instrument equally adaptable for transparent and opaque preparations. All types of illumination are thus involved: transmitted, both direct and dark-field; reflected, both vertically and inclined; and with facilities for polarization in connection with all these means of illumination. Magnification must range from very low to the highest attainable. It was our aim effectively to combine these several functions without departing from highest standards for each function. We desired, furthermore, to provide for easy adoption of all essential accessories now known, and, so far as possible, for those that may be developed hereafter, of whatever kind. The word accessories is here used in the broadest sense, to include everything from

[2] "A Precision High Power Metallographic Apparatus," Trans. Am. Soc. for Steel Treating **21**, 1112–1134 (1933).

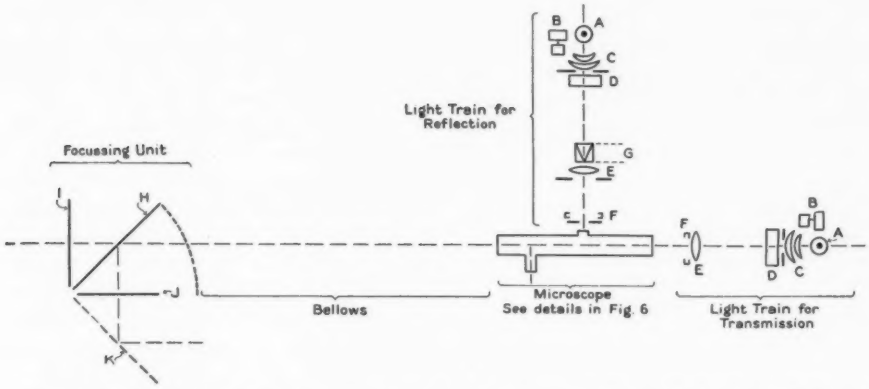

FIG. 1. Diagrammatic plan of instrument. *A, A*, light sources; *B, B*, ventilating fans; *C, C*, condensers; *D, D*, cooling cells; *E*, auxiliary condenser; *F, F*, filter receptacles; *G*, polarizing prism; *H*, swinging first surface mirror; *I*, focusing screen or plateholder; *J*, supplemental focusing screen; *K*, supplemental mirror.

means of illumination to the various optical elements. This meant, among other things, ample "room" everywhere, yet without taking liberties with tube length and other focal distances. These several considerations were the most fundamental in determining the design.

Simplicity of light path

It was our aim to interpose into the light beam, especially after this has acquired the desired image, the fewest possible optical elements; since each unnecessary lens or prism, with its unavoidable imperfections of one kind or another, inevitably aggravates the cumulative impairment of the final result. We therefore adopted a layout taking fewest liberties with the natural rectilinear quality of light. Inasmuch as this instrument was designed primarily for photography and secondarily for visual examination, and since in any event the tolerance of the eye is greater than that of the photographic plate, we provided a straight-line axis from object through the microscope to the sensitized plate. One consequence was the forced elimination of the popular inverted type of reflecting microscope. Since the length of the instrument imposed a horizontal axis, another consequence of the simple light path was to give the stage a vertical position; this we regard as inherently less satisfactory in certain mechanical respects than the horizontal stage, but a compromise which other considerations justify and which is not without certain advantages of its own. The general scheme of arrangement is shown in Fig. 1.

Standards and materials

The standards of tolerance and precision required for the highest type of instrument making are followed throughout. Stability and freedom from flexure are attained by following the best principles of structural design. Save where ball bearings are used, parts in frictional contact consist of dissimilar metals, generally gray cast iron and bronze or steel and bronze; the castings were seasoned for months before being finally shaped; and the bearing surfaces were either hand scraped or lapped to extreme flatness. The stage assembly, the sub-stage unit and the photographic ocular mounting, each moves along the main axis on broad dovetailed ways and is provided with an adjustable gib to insure permanently the proper closeness of fit. Utmost care was given to the making of the fine focusing drive, discussed later.

Stability

Absolute and permanent stability of every part, from foundation to specimen and photographic plate, implied durable materials, massive construction, absorption of vibration, and rigid clamping, as well as highest precision of fabrication.

The room in which the microscope is installed has a 25 cm monolithic concrete floor poured

directly on the sub-soil. Resting on this floor, six rubber-in-shear marine-engine mountings to absorb foreign vibrations support a 360 kg steel foundation bed of heavy channel section 38 cm wide, and having in plan the shape of a squatty inverted T corresponding to that of the instrument itself; the long member measures about $3\frac{1}{2}$ m ,the short one 85 cm. The general scheme of foundation support is indicated in Fig. 2, where the rubber mountings are shown at (1) and the main channel bed at (2). Bolted to this rubber-floated foundation are the uprights that carry the beds. Three of these uprights (3) support the two light-train beds (4), and the other two (5) carry the main optical bed. The first-named uprights are adjustable by large leveling screws at their bottoms so that they can be brought into exact alignment or perpendicularity with the main optical bed carried by the larger uprights.

The heavy castings that constitute the light-train beds are each 1 meter long. The one for transmitted light is mounted on rollers, for reasons later mentioned. The main optical bed (6), which supports the microscope and the camera, is a truss-shaped casting $2\frac{1}{2}$ meters long, stiffened by integral cross webs at intervals of 30 cm; it weighs 150 kg. Independently of the other two beds, this important member is individually cushioned from its supporting uprights by a 4-point suspension on built-up pads of sponge rubber held by compression in container-bosses (7) on the uprights; three layers of one texture of the rubber alternate with three layers of a different texture, so that a wide range of vibration period is damped out.[3] To compensate for progressive and perhaps differential compression of the rubber pads, the main optical bed can be easily leveled from side to side by leveling screws

[3] These rubber pads can be removed and new ones inserted with little difficulty. But if we find that they lose resiliency too quickly, we shall probably replace them by the rubber-in-shear type of mounting.

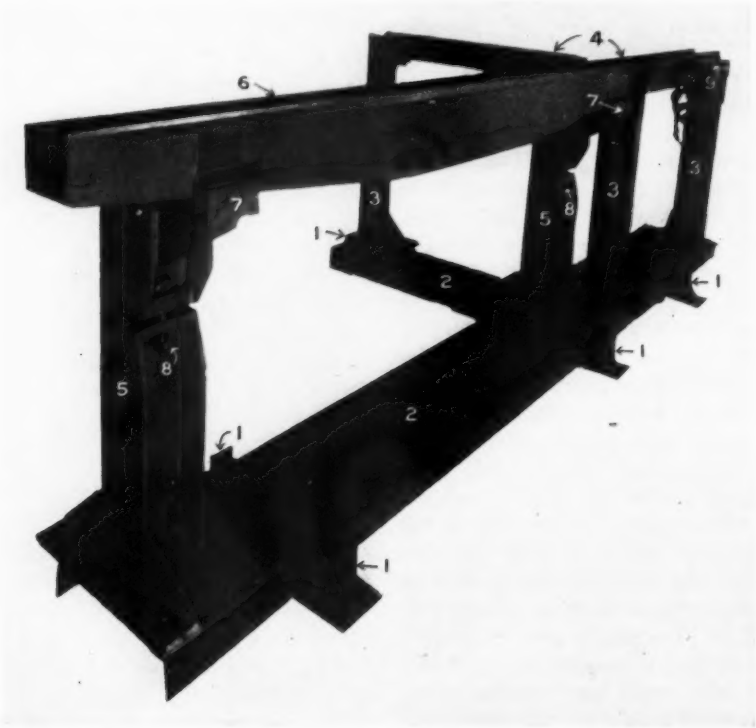

FIG. 2. Optical beds and manner of support (some distortion in photograph).

at each of the containers on the far side of the bed (hence not shown in Fig. 2). And in order to level the bed from end to end at the exact elevation of the light-train beds, adjusting screws (8) that operate vertically are located on the uprights and the proper position of the bed then definitely fixed by heavy clamping bolts.

The entire instrument, including foundation bed, superstructure and optical parts, constitutes a mass of approximately 900 kg bearing on the concrete floor. This is the mass to which external vibrations would have to be imparted through the rubber-in-shear mountings. Professor L. D. Leet, of the Harvard Seismological Station, very kindly tested the stability of the main optical bed thus supported. A small sensitive seismograph, having a magnification factor of 1000, detected the arrival of extremely faint vibrations at the optical bed when a 35 kg weight was dropped from a height of 80 cm on the concrete floor 2.5 m away from the instrument; but there was no indication of *differential* vibration from end to end of the bed. If directly on one end of the main optical bed a sharp blow be struck while an image at 3000–5000 magnifications is being watched on the focusing screen by a magnifying lens, the image is seen to vibrate through an amplitude of something less than 1 mm; but this subsides to zero in about a quarter or half second. Nothing whatever of this sort has ever been seen that originated from external vibrations.

The linoleum-surfaced false wooden floor which bridges the channel section without touching either it or the uprights permits no vibrations from movements in the room to reach the instrument. The supporting uprights are located not directly under the main optical bed, but back of its center line, and support it by cantilever brackets; this is to remove them from the reach of possible jar by the operator's feet. The work-bench (24) extending conveniently along the entire front of the instrument guards it from unintentional shock by being supported independently on the false floor. The bench is linoleum-topped and provided with a raised bead at the edge so that small articles will not so easily be accidentally brushed off; as shown in Figs. 3 and 4, it is narrowed opposite the microscope to permit convenient proximity of the observer to the visual ocular.

Stable placement of the various optical parts is attained as follows. Each of the three beds is finished with a V-rail on the near side and a flat rail on the far side, affording a base 15 cm wide. Horizontal slots milled in the inner side of these rails give means for the same rigid clamping of all parts carried by the beds as is accomplished with the tail-stock of a lathe. The optical axis is brought as close to the bed (19 cm) as size of the maximum photographic plate adopted (18×24 cm) and adequate mounting of the working parts would permit. Each support, whether for lamp-house, condenser or camera, is a sturdy webbed casting of such generous contact along the length of the bed as to insure positive rigidity when clamped. Two of the larger supports are indicated at (10) in Fig. 4, and smaller supports and the method of clamping may be seen in Fig. 5.

Freedom from dimension-change with temperature is favored by the fact that the room is in the heart of the building and thus windowless, being provided with forced ventilation. A metal-bulb thermometer clamped on the optical bed immediately below the microscope registers by remote connection through a dial on the instrument dial-board (20) located directly in front of the operator; maximum temperature changes from day to day rarely exceed 2°F and take place at a rate far too slow to affect focus. Excess heat from the light sources is virtually eliminated by the nature of the lamps (later described), by fan-ventilation of the lamp-houses, by the introduction of cooling cells, by independently mounted irises that prevent light from striking the metal of the microscope, and by complete protection of the most critical focusing parts within heavy metal housing. Where the breath from the observer at the visual ocular would strike the microscope base-castings and cause warming or condensation, a protecting layer of cork is cemented.

In conjunction with the massive build of the microscope proper, described hereafter, the foregoing combination of design and construction proves to have achieved the stability and constancy desired. After a continuous exposure of as much as three hours at magnification up to 6000 diameters, removal of the plateholder and reinsertion of the focusing screen invariably shows the focus to have remained exactly as at

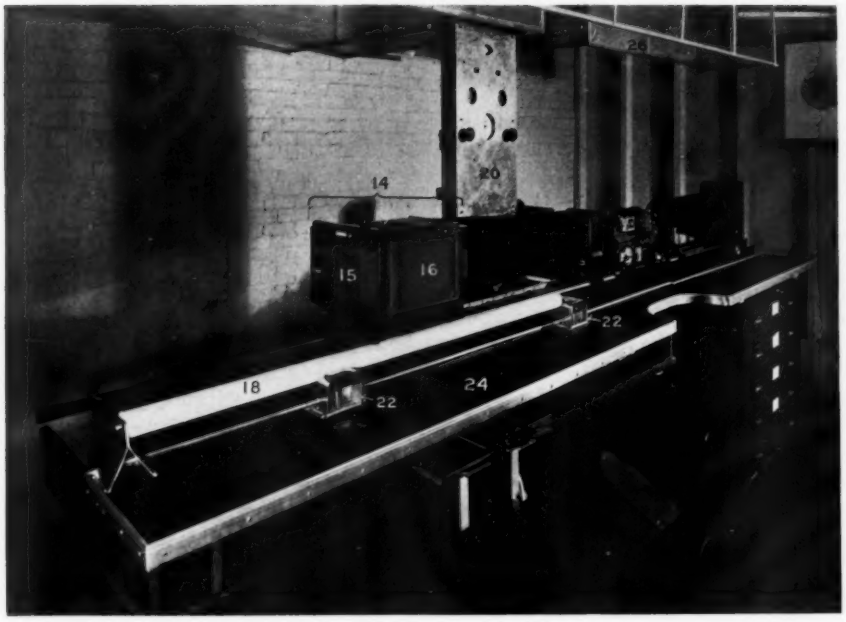

FIG. 3. General view of instrument from rear.

the start; and the negative is completely free of vibration effects. Apparently no more could be desired in these respects. The materials and workmanship incorporated encourage the belief that this satisfactory performance will persist through many years of use.

The entire instrument is protected from dust and damage when not in use by a skeleton aluminum structure on which is stretched "Celloglass"; the whole suspended by counterweighted cables and roller guides so that it may be readily lifted out of the way when the microscope is used. A part of this cover is shown at (26) in Fig. 3.

Adjustability

Complete adjustability of every essential part seemed to us indispensable. This is in direct contrast with the common present-day policy of "perfect adjustment at the factory"—and essential unchangeability thereafter. On our instrument, therefore, distances, angles, centering, etc., are in practically every instance subject to controlled adjustment and to rigid clamping or other fixation when properly adjusted.

The axis of the entire microscope assembly is fixed by the center of the sleeve that receives the photographic ocular, since this has an unchangeable position with respect to the ways of the optical bed. Correct collimation is achieved by means of a pair of test units (Fig. 5) that can be placed anywhere along any of the three beds. These units are provided with sliding points and also iris diaphragms fixed in the exact axis of the microscope; thus centering may be tested either mechanically or by a tiny beam of light passing through the two irises. Each of the test units carries a precision spirit level for checking the horizontality of the optical beds.

The light-train bed for transmitted light can be moved toward or away from the microscope on carefully mounted V-rollers, and then clamped in place (see (9) in Fig. 4); this facilitates the maintenance of critical illumination with objectives of different focal length which call for different positions of the stage. Such adjustment is not needed for the vertically reflected beam. The light-bed for the reflected beam can be swung through a considerable horizontal arc, in case it be desired to bring the

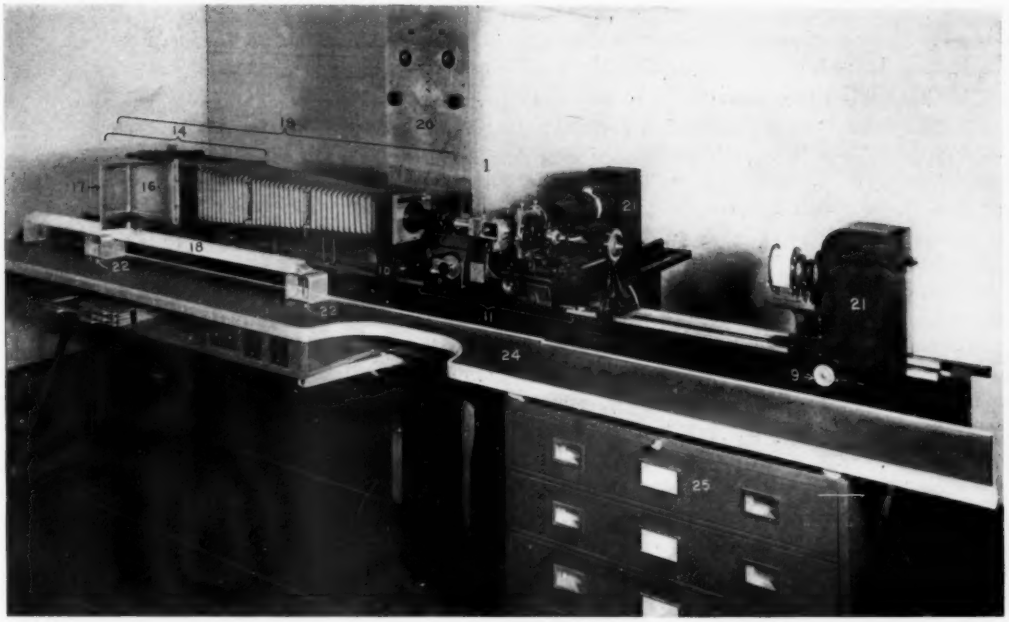

FIG. 4. General view of instrument from front.

incident beam into the vertical illuminator at a different angle to the optical axis than the usual 90°.

ILLUMINATION

Light sources

The design of the beds is intended to permit the employment at will of any of the sources of illumination thus far devised, whether carbon or metallic arc, incandescent filament, Point-O-Lite, mercury vapor, or various special forms of monochromator.

We decided on the low voltage ribbon filament incandescent lamp of which the satisfactory and dependable character[4] had been proved by us through extended prior use. Through the cooperation of the Nela Research Division of the General Electric Company, special bulbs were made with the flat tungsten filament of large dimension and of such shape as to project through to the microscope the minimum of nonusable light; they burn at 3.5 volts and 50

[4] Our experience with the arc lamp has not been good; unsteady mechanical feed, flickering and wandering of the arc lead to nonuniform illumination over the negative and erratic exposure times.

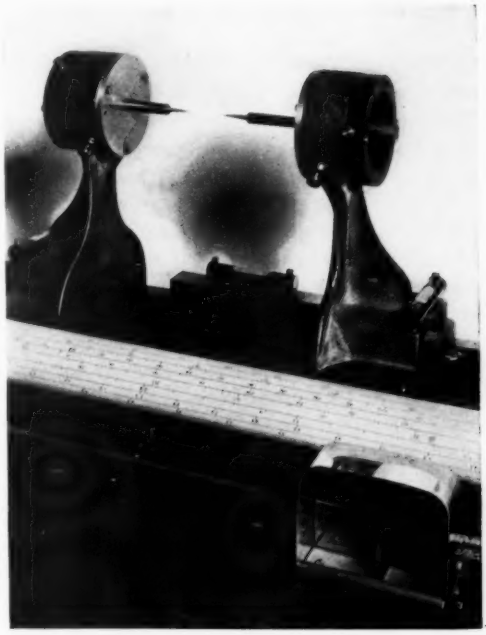

FIG. 5. Test units for collimating, also focusing switchbox and magnification scale.

amperes. The motor-driven fan, carefully balanced for high speed and rubber-mounted on the lamp-house to reduce vibration, directly cools the lamp and the adjacent condenser; it is so wired as to operate only when the lamp is burning. Into the lamp circuit is also shunted a telechron motor geared to a Veeder counter on the dial board; thus the use of each lamp is automatically metered in hours so that the total life of these somewhat experimental bulbs may be recorded. Except in one respect these lamps have proved highly satisfactory. For transmitted work, and the range of magnification there needed, they meet every need. But for vertical illumination at highest powers they are disappointing. For at very high powers nearly monochromatic light is needed to give good images, even with the best apochromatic lenses available to us; and when the dense filters necessary to attain this from the tungsten filament are used, there is inadequate illumination at these high magnifications for accurate focusing, and the exposures are inconveniently long. This situation is further aggravated when the polarizing prisms are used.

We therefore installed in the lamp-house for reflected illumination an extremely simple miniature mercury-arc (high pressure capillary) which operates at 85 watts, General Electric Company's type H-3.[5] It is admirably steady, and by appropriate filters[6] gives essentially monochromatic light in blue (λ4358), green (λ5461), or yellow (λ5770–5791) of sufficient intensity for sharp focusing at highest powers. Under these conditions, exposures in the green on Wratten "metallographic" plates average about 90 seconds for 3000 magnifications. At some possible sacrifice of life of these lamps by increasing the voltage we expect to gain still greater intensity. This type of lamp, like the carbon arc and the Point-O-Lite, requires exposure to be made by means of a shutter or equivalent mechanism, and is consequently in this respect less convenient than the ribbon-filament bulb with which exposure can be accurately controlled by the switch.

Just outside the vertical illuminator, a decenterable rotatable iris provides any degree of obliquity of illumination from any azimuth. Receptacles for filters (see Fig. 1 at *F*) in both the light trains are located within easy reach of the operator when seated at the microscope. Switches for the control of either lamp and also for the general illumination of the room are similarly convenient, just below the front edge of the work bench. The lamps for reflected and for transmitted light may be burned simultaneously, and by appropriate iris diaphragms can be so balanced as to relative intensity that a "thinned polished section" or a "polished thin-section" may be effectively studied and photographed as to both transparent and opaque components simultaneously.

Polarization

Polarizing prisms[7] for both transparent and opaque work were incorporated as an essential part of the instrument, not hitched on as an afterthought or stingy-sized appendage. We are glad to note that the latest standard metallographic instruments have greatly improved their prior practice in this respect. Ahrens prisms were selected as being of higher effectiveness than the more common Nicol type. All three are of large cross section in order to minimize light loss; for example, the polarizer for the beam to be reflected measures 20×20 mm; the analyzer, 18×18 mm. Each can be moved readily into or out of the light beam, and can be rotated to any azimuth through a 90° range in graduated mounts.

By means of dovetailed slots located on either side of the analyzer position, so-called correcting or stigmatizing lenses may be introduced in proper position, in order to obviate the astigmatic effects and the changed magnification and focus occasioned by the analyzer when not in a parallel beam. (See Fig. 6 at *F* and *I*.) The rear one of these lenses, i.e., that one farther from the microscope stage, when used alone, serves also as the "telescope lens" that permits the use of the

[5] Cf. Barnes and Forsythe, J. O. S. A. **27**, 83 *et seq.* (1937).

[6] We ordinarily use Corning glass filters.

[7] None of the new crystal film polarizing plates which we have tried out thus far proves acceptable for the level of precision demanded by our instrument; the principal difficulties experienced are nonuniform polarization over the field and excessive light absorption. It is hardly to be doubted, however, that this interesting method of obtaining polarized light will soon be further perfected. It will then be especially useful for such purposes as ours, not only because of relative cheapness (which will encourage the use of adequately large cross sections) but even more because of their economy of tube length.

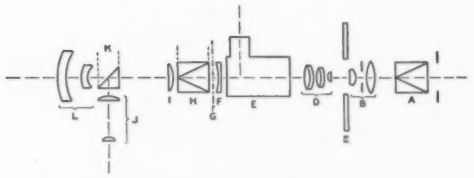

FIG. 6. Schematic arrangement of microscope assembly. *A*, polarizing prism; *B*, sub-stage condenser; *C*, mechanical stage; *D*, objective; *E*, vertical illuminator; *F*, *I*, stigmatizing lenses; *G*, slot for test plates; *H*, sliding analyzing prism; *K*, sliding reflecting prism; *J*, visual ocular; *L*, photographic ocular.

recently-developed "infinity-corrected" objectives.

THE CAMERA

The camera assembly comprises a light-tight connector at the photographic ocular, the bellows, and a box-like focusing unit, all carried on appropriate sliding supports. In order to avoid the annoyance, when change of ocular is desired, of having to unclamp the front end of the bellows support and slide it back and then restore it, the connector between bellows and ocular in this instance is a split tapering cylinder, of which the upper half is hinged, so that it may be opened back (see (33), Fig. 8) for easy access to the ocular, but is light-tight when closed (see (33), Fig. 7).

The focusing unit embodies the convenient reflex principle, adapted from Reichert, by means of a swinging mirror bearing a first-surface evaporated film of "Chroluminum" and mounted so that it moves very easily through the arc from the 45° to the zero position. It is held thus definitely "open" or "closed" against leather padded stops by a permanent magnet, mounted on the outer edge of the mirror frame, which just fails to touch a steel block permanently fixed at either extreme of the swing; the "pull" of this magnet when thus in closest position to its armature suffices to prevent the mirror from accidentally swinging out of azimuth. The prime reason for incorporating this means of focusing was to eliminate the possibility of injuring the focus by the jarring incidental to the later steps. We have found in practice, however, that all the supports are so sturdy and rigid and the optical bed so massive that focus is not disturbed by the jarring that attends insertion and removal of focusing screen and plate holder in the conventional manner. We therefore resort only rarely to reflected focusing. But we find it convenient to use the reflecting mirror in conjunction with a second 45° mirror (see (17) in Fig. 4) outside the ground glass so that the image may be seen while the observer is seated before the microscope proper. It is thus possible, before moving back to the focusing screen, to adjust centering of the subject (and preferable azimuth, if polarized light is used), also to attain approximate focus for low power lenses that take no ocular (which focus at a very different setting from that used at the visual ocular). Final perfection of this focus is attained on the focusing screen at the rear after swinging the mirror to the closed position. The focusing unit is seen at (14) in Figs. 3 and 4 and is shown diagrammatically in Fig. 1.

The maximum bellows extension from the photographic ocular to the rear end of the main optical bed is about 2 meters. The bellows is held in position at any extension by intermediate supports provided with rollers where they rest on the optical bed.

Magnification, for any given optical combination and bellows extension, is read off directly on a graduated scale by an index pointer attached to the focusing unit at the plane of the photographic plate. The several scales are ruled on a white "Duco" enameled cylindrical tube mounted to rotate on its axis alongside the part of the optical bed corresponding to the bellows extension. There is room on the cylinder for 25 such scales, each graduated in terms of magnification diameters, at the different extensions, for a given useful objective-ocular combination. The scales were initially calibrated against engraved stage micrometers, checked (for reflected light) by a Rowland diffraction grating having 14,438 lines to the inch (interval, 1.758 microns). This automatic means of ascertaining or determining magnification is a great convenience and time-saver. The set-up is seen at (18) in Fig. 4, also in Fig. 5.

The dark slide of each plate holder carries spaced marks that permit the making of a series of uniform "strip exposures" to test focus, exposure, or filter. But still more useful, when serious experimentation is desired, is the Zeiss

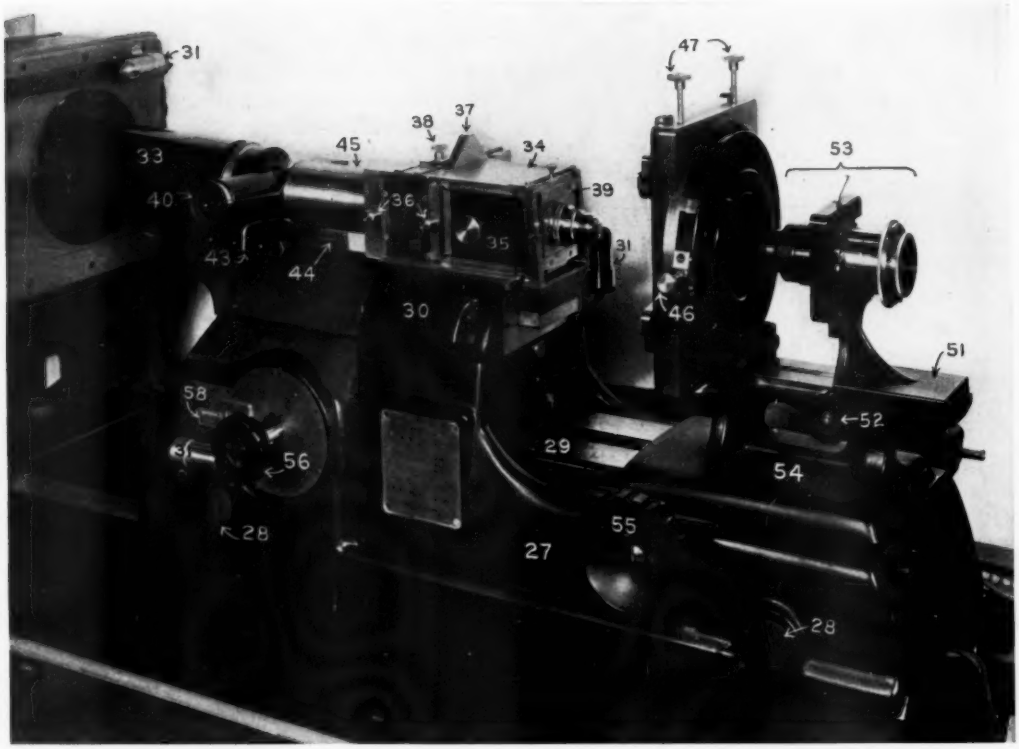

FIG. 7. View of microscope from right.

"Multiplicator," interchangeable with our regular plateholders and focusing screens; this allows the making of a series of strips (of predetermined width) duplicating the *same portion* of the subject. A series of such strips, taken to show differences in focus, is reproduced as Fig. 11.

THE MICROSCOPE

General design

The microscope proper, represented schematically in Fig. 6, departs widely from conventional design. Its general appearance is shown in Figs. 7 and 8, to which apply the key numbers in parentheses used in this section of the description. The base of the microscope (27) is a massive casting 45×15×15 cm in dimension. It is accurately fitted at its four corners to the main optical bed, along which it may be slid and firmly clamped (28) at any desired place—ordinarily it is fixed at the extreme right end of the bed. On the upper surface of the right-hand half of the casting are machined the ways for moving the stage assembly (29). Bolted to the machined surface of its left-hand half is a supplemental casting (30) that carries the tube assembly and encases the ball-bearing ways for the fine adjustment.

Tiny electric bulbs in protecting shields (31) are so placed as to illuminate the various scales. They are controlled from a convenient switch (32) on the base-casting and are fed through sliding contact with leads on the main optical bed, regardless of where the microscope may be placed.

The tube

The major portion of the microscope tube is of large rectangular section (34), bolted directly on the machined surface of the supplemental casting. It carries at appropriate positions a series of horizontal dovetailed slots for the

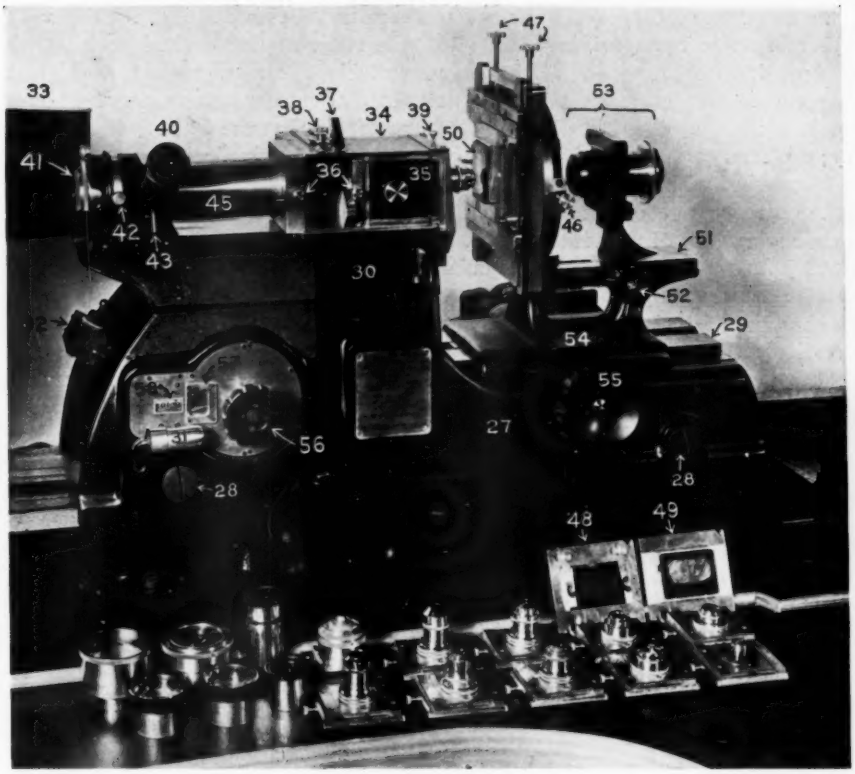

FIG. 8. Microscope—also representative optical parts and specimen holders.

interchangeable reception of the vertical illuminators (35), and the stigmatizing lenses (36); a slide in the 45° position (37) ahead of the analyzer accommodates a bi-quartz wedge or other test-plates, all of large size (20 mm). The analyzer, in an enclosed housing, may be slid into or out of the field and may be rotated through 90° by means of a knurled and graduated knob (38) geared to the analyzer shell.

Each objective is screwed into an independent mount (39) that slides into a vertical dovetail at the front (right-hand) end of the tube; each mount provides for permanent centering of its objective by adjusting screws so located as not to be damaged or disarranged if the unit should accidentally be dropped. The mounts are sturdily built and accurately scraped so that each slides home precisely and then remains absolutely rigid during even the longest exposures; they are of generous dimension so that they may accommodate the conventional "Society" objectives and the larger "Ultropak" or "Epi-condenser" objectives; bushings allow the use of the smaller "infinity-corrected" objectives; they will also take regular photographic objectives up to 4 cm diameter, either in the usual position or in the reversed position recommended by Loveland.[8] The knob for convenient handling has a number engraved on its top surface for ready identification when the objectives are filed in their storage drawer. Several objectives in their sliding mounts are shown on the work-bench in Fig. 8.

The two oculars, respectively for visual inspection (40) and for photography (41), are supported by a bronze casting. The tube for the visual ocular can be swung through a considerable vertical arc and held by friction, so that the individual operator may select that angle of viewing he finds most natural. Provision for

[8] J. O. S. A. **26**, 224 (1936).

bringing the illuminating beam from one ocular to the other is made by a prism that slides smoothly through a fixed range in guides within the casting and is controlled by a convenient handle (43) while the visual ocular remains stationary; this arrangement was chosen because the beam is switched with less chance of jarring than by the customary means of attaching the prism to the sliding tube of the visual ocular. A slot located just ahead of the visual ocular accommodates whatever modulating glass or filter may be desired for visual study only. The tube for receiving the photographic ocular is of such size that it will accommodate the largest Homal (42 mm diameter). Ampliplans and the smaller Homals, as well as the conventional 23.3 mm eyepieces are used in reducer-bushings that also fix the given ocular at its appropriate position to provide the desired mechanical tube length. By a spiral mechanism controlled by a knob (42), the tube length can be varied through a 6 mm range to compensate for different bellows extensions.

To permit the use of objectives corrected for different tube-lengths, as well as the optional use of the stigmatizing lenses, the ocular-mount is made to slide in the direction of the optic axis on dovetailed ways (44) of broad gauge. A hollow cylinder that readily snaps in place by a simplified bayonet catch provides a light-tight spacer between the ocular carriage and the fixed portion of the microscope tube. For each of the most customary tube lengths of the principal objective makers there is a cylinder of appropriate length, engraved to show the particular tube length for which it is intended. The spacing cylinder shown at (45) is that for 215 mm when the two stigmatizing lenses are used. The appropriate spacer in conjunction with the rear correcting lens permits the use of infinity-corrected objectives, as already mentioned.

Mechanical stage

The revolving stage has been sturdily rebuilt on the basis of the beautiful ball-bearing mechanism recently introduced by Leitz in their best petrographic microscopes; the result combines stability[9] with an admirable smoothness of rotation. A clamp (46) of the floating type locks the stage at any desired position without detectable decentering even at highest powers. Angular graduations are engraved on a flange so protected that the figures will not be blurred by wear. The mechanical cross motions, operated at 90° by knurled knobs seen at (47), have travel of 25 and 40 mm, respectively, registered by graduated scales. These motions are accomplished by hardened steel carriages of the pre-loaded ball-bearing type, and in all respects are highly satisfactory save that the present operating screws must be changed to a pitch that provides less rapid travel.[10] The specimen remains exactly centered with respect to the optical axis regardless of the position given it by the mechanical motions; and by appropriate rotation of the stage, the travel can be induced by either screw in any desired azimuth, whether horizontal, vertical or oblique.

Special specimen holders are provided respectively for thin (transparent) and for polished (opaque) sections. One of each of these kinds of holders is shown, at (48) and (49), respectively, in Fig. 8. Both types have the base of identical shape and dimensions and provided with a bevelled notch and two bevelled flat places; these can be readily engaged under three hardened stops on the face of the stage, one of which is an eccentric cam; in this way any holder can be rigidly clamped snugly against the stage plate in a standard and invariable position. The holders for transparent preparations accommodate thin sections of conventional dimensions held by spring clips in such way that a given section can be instantly returned to exactly the same position in the holder. An analogous means permits the same to be done with the polished sections in the rectangular molded Bakelite mounts which have long been standard in our laboratory.[11] As a consequence of this arrangement, for any specimen, which has once been fixed in a holder and clamped on

[9] Although the rotating mechanism is itself entirely satisfactory, the mount of the stationary ball-race is not sufficiently massive to escape occasional slight spring-like deflection while the stage is being manipulated—the result is a detectable temporary impairment of focus when highest powers are used. This is to be remedied by the addition of stiffening supports. In the Ottawa instrument, the appropriate parts are heavier and no trouble is experienced.

[10] For the Ottawa microscope, the screws are of slower pitch.

[11] Cf. "Technique in Mineralography at Harvard," Am. Mineralogist **22**, 496 (1937).

the stage, any desired point brought to the center of the field of view by the mechanical motions may have its coordinates read from the scales and permanently recorded together with the identifying number of that specimen; at any later time, that specimen, when fixed in a holder, clamped on the stage and set to the same coordinate readings, invariably discloses the original point in the center of the field of view. Moreover, since the coordinates and the position of the origin are calibrated to the same standard as adopted throughout our laboratory, it follows that any feature studied (and coordinates recorded) by any worker on his own microscope may at any later time be taken to the microcamera and immediately be brought into the field of view for photographing; this can be done even when an objective of high power is used on the microcamera from the beginning. The annoying and time consuming hunt for a place or feature previously seen or studied is thus entirely avoided.

The holder grips the specimen, whether thin or polished, in such way that nothing projects beyond the surface of the specimen. (See polished section in its holder clamped to stage at (50) in Fig. 8.) The result is that an objective of even the shortest working distance can scan the *entire* surface of the specimen without the danger of injury by having metal parts jammed against the tip of the objective by the mechanical motions when searching near the edge of the specimen, as is liable with the inverted microscope. This method of mounting so that the surface of the specimen projects beyond all its surroundings is especially helpful also in attaining approximate focus when oil immersion objectives are used; by placing the eye in the extension of the plane of the specimen surface, one can watch the space between specimen and objective diminish as the coarse focusing mechanism is operated, and thus can stop when the distance between the two is approximately correct. Our actual procedure when focusing with oil immersion objectives is to bring specimen and objective by the coarse focusing mechanism so that they *almost* touch but still show a narrow slit of light between them, in which position they are "inside" the focus (i.e., too close together); now, transferring the eye to the visual ocular, the fine focusing mechanism is operated, in the direction that *separates* the objective from the specimen, until correct focus is obtained. This procedure saves time and avoids possibility of injury to lens or specimen.

Sub-stage

The sub-stage equipment for transmitted light, consisting of condensers, two iris diaphragms and an Ahrens polarizer, are supported on a special bronze casting (51) that can be moved along the light axis on dovetail ways by a large knurled knob (52). The assembly is shown at (53); in Fig. 8 only the low power condenser of N.A. 0.25 is shown; the auxiliary slip-on condenser for higher powers bringing the N.A. to 1.00, is shown in Fig. 7. The sub-stage equipment as a unit, or any part of it, may be readily removed from the light beam.

Coarse focus control

Coarse focusing is accomplished by moving the entire stage and sub-stage assembly along sturdy longitudinal ways by means of a spiral-cut rack and pinion of generous size. The casting that supports the assembly is seen at (54). By this arrangement the coarse focusing has nothing to do with the delicate mechanism for fine focusing. The stage has sufficient travel along the axis to permit the use of objectives ranging from short mounted oil immersion lenses of the shortest working distance to regular photographic lenses having a maximum focal length of 10 cm. The coarse focusing knob (55) is remote from that which controls the fine adjustment (56), besides being much larger, so that by no chance can damage arise through inadvertently grasping the wrong knob when working in the faintly lighted room.

It is sometimes desirable to operate the coarse focus when the operator is at the focusing screen of the camera. In order to achieve this, the coarse focusing device may be actuated by motor, controlled from either of two forward and reverse switches located at places on the workbench conveniently in reach either when the operator sits before the visual ocular or at any position of the focusing unit along the length of the bellows draw. The motor drive operates through a self-actuated magnetic clutch engaged

only when the focusing switch is closed; thus there is no conflict with the hand focusing mechanism. The present clutch, of the jaw type, tends to cause a slight lunge of the stage, either forward or backward, at the instant of engagement, and will be changed for a smooth-faced friction clutch.[12]

Fine focus control

In our experience and belief, the means for insuring perfection of focus at high power have been surprisingly neglected.[13] Two fundamental defects are encountered: the slow motion is not slow enough, and there is always lost motion or back-lash. Each is fatal to the invariable attainment of the best possible focus; and each augments the shortcomings of the other.

It is to be remembered that at a magnification of 2000 diameters the intensity of light on the focusing screen (even disregarding losses in the microscope system) is only 1/4,000,000 of that which comes from an equal area of the object. In consequence, even with the most powerful light sources, it is as yet almost inescapable that truly high power focusing is done with an insufficiency of illumination. The eye, therefore, is taxed near to its limit in watching for the perfection point as the mechanism is operated in the direction of improved focus; and the longer this strain is endured, the more rapidly fatigue breaks down the eye's sensitivity. Moreover, with increasing magnification, the *apparent* speed of the slow motion increases. Finally, it is to be noted that the *depth of focus* of high power objectives appears, by actual test, to be substantially less than indicated by computation from the currently accepted formulae.

With instruments of conventional design, therefore, in focusing at magnifications of 1000 or 2000 upward it is common experience to overrun focus, not only once, but forwards and backwards repeatedly, because of this vicious combination of too fast motion and back-lash. And although gradually the error is reduced, by about the time one feels he has practically "cornered" perfect focus, his eye has lost its sharply critical quality; and after a rest he is likely to overrun badly again and have to begin anew. At last he capitulates to an "acceptable" compromise.

In the design of our instrument, therefore, we have nowhere else concentrated so much attention as in the slow motion mechanism, since it is obvious that no lenses, however perfect, can be counted on to yield their best unless they can be set invariably at exact focus.

As the first step toward the attainment of our ideal, we have adopted the recently revived application of the old principle that the fine adjustment should be called upon to do the least possible amount of work. In most instruments, this delicate mechanism is required to move not only the entire microscope tube with all its appurtenances including objective and ocular, but also the whole mechanism for coarse focusing. Such design is obviously fallacious.

In our instrument, the fine adjustment moves only a minimum mass composed of the objective and the horizontal carriage on which the objective rides. This carriage, consisting of a hardened steel plate and a forked upright into which the objective holders fit, travels in preloaded contact on four rows of balls following the general scheme used by Leitz. To prevent any possible cramping through slightest non-parallelism of the grooved races, only one row of the balls controls the longitudinal alignment of the carriage, the other three rows have flat races on one side so that the carriage "floats" upon them. The cross section seen in Fig. 10 makes clear this method of mounting. The system actually proves to combine high rigidity with a very perfect freedom of linear motion of the carriage.

Such method of attaining fine focus has the theoretical disadvantage of changing the tube length from that for which the objectives are corrected; but since in our case the maximum travel of the objective is only 1 mm in either direction from a zero starting position, and since in practice only a small fraction of that available 1 mm has to be used in order to

[12] This improvement has been made in the Ottawa instrument; also, the entire stage assembly moves on preloaded ball-bearings instead of sliding on dovetailed ways as in our instrument—this gives smoother motion.

[13] The extremely interesting paper by Dr. H. F. Kurtz on "Mechanical Design of the Microscope" (presented at the same symposium as ours but unfortunately not yet published) illustrated a wide variety of fine focusing mechanisms. These seemed in every instance to reveal on the part of the designers a contentment with the existing *standards* of fine focus control even if ideas differed as to means of attaining those standards.

attain perfect focus, it follows that the deviation from the ideal tube length is entirely negligible, and no impairment of the image on this account can be detected. Motion of the carriage and attached objective is accomplished through an extremely simple but sturdy and highly accurate train comprising a worm : worm-wheel and screw : nut (see Fig. 9 for details). The fine focusing knob (56) is attached to the shaft of the hardened steel worm—enclosed in the housing (59), Fig. 9—which has such pitch with respect to the diameter of the bronze worm-wheel as to occasion here a 250 : 1 reduction. The face of the worm-wheel (60) is concave and its teeth as well as the threads of the worm are deep (angle 29°) so that an unusually large aggregate surface of contact exists between the two, but mirror finishing of both and special provision for oil mist lubrication and dust exclusion insure minimum friction and maximum life. The worm is mounted on trunions ((69) of Fig. 9) at one end and held snugly in spring contact against the worm-wheel so that neither end-play, back-lash nor binding is possible at this junction.

The worm-wheel is rigidly fixed to a steel shaft (61) mounted parallel to the microscope axis in burnished bushings. On the middle portion of this shaft is cut a screw of 0.5 mm pitch, 70 threads long. An accurately adjustable split nut (62) of bronze, covering 62 of the threads of the screw, is provided with a torque arm (63) of which the cylindrical burnished end slides between two fixed parallel plates of glass so placed that rotation of the nut is zero when the screw is turned in either direction. The rear end of the screw shaft carries a plate of polished sapphire (64) which bears against the polished end of a steel rod to take end-thrust. On a boss of the nut, a hardened, polished face (65) transmits the forward motion of the nut to the sliding objective carriage—at (70) in Fig. 10—thereby compressing a helical spring (71) of such stiffness as substantially to exceed the frictional resistance of the entire gear-train. Thus as soon as the reverse direction of the focusing knob is started, the spring causes the objective carriage to follow instantly.[14] Back-lash is thus reduced to a vanishing quantity.

The 250 : 1 ratio of the worm : worm-wheel pair, in combination with the 0.5 mm pitch of the screw, gives the result that 500 turns of the focusing knob moves the objective 1 mm. On the shaft which carries the focusing knob and worm is a dial (its housing is seen almost edge-on at (66) in Fig. 9) of such diameter that 200 radial divisions engraved near its periphery are spaced exactly 1 mm at the outer edge. Thus each dial division stands for 0.00001 mm travel of the objective (i.e., 1 division = 100A). Every fifth division on the scale is extended for accent, and every tenth one is numbered, up to 100 and then repeated once more to 100. As all of the dial is concealed in a housing except a small section opposite the fixed zero line exposed through a glass window, no confusion arises from the graduation into two 100's. The worm-shaft is so geared (67) into a 5-column Veeder counter (68) that each 10 divisions on the dial causes a change of *one* in the right-hand column of the counter. The windows for dial and counter are shown (57 and 58) in Figs. 7 and 8. It is thus possible to read the position of the objective throughout its entire range of travel; a reading of 100,000 represents the mid-position or starting point of this travel, 200,000 represents one extreme and 000,000 the other extreme.[15]

Inasmuch as the entire gear train involves the screw principle, with no use of levers, each dial division represents exactly the same amount of movement of the objective; the mechanism is thus a highly reliable micrometer. The worm and screw have been cut on a lathe having a master screw highly corrected for errors of all kinds, and every part of the mechanism has been made and assembled with a precision of the same order as required for a grating-ruling engine. Interferometer tests of this focusing motion are recorded under the heading "Performance and Possibilities."

[14] In our instrument the objective carriage is positively driven, against the spring, toward the object. This was a mistake. In the Ottawa instrument, the relative positions of end-thrust bearing and helical spring are reversed, so that the objective is carried toward the object only by the spring and the *positive* drive is away from the object; this greatly reduces the danger of damage if focus should be accidentally overrun and objective pressed against object.

[15] The end of the threads of the screw would not be reached until the readings changed by 50,000 units more in either direction; and even then the nut would not jam, since the threaded portion of the screw is of larger diameter than the plain shank at either end.

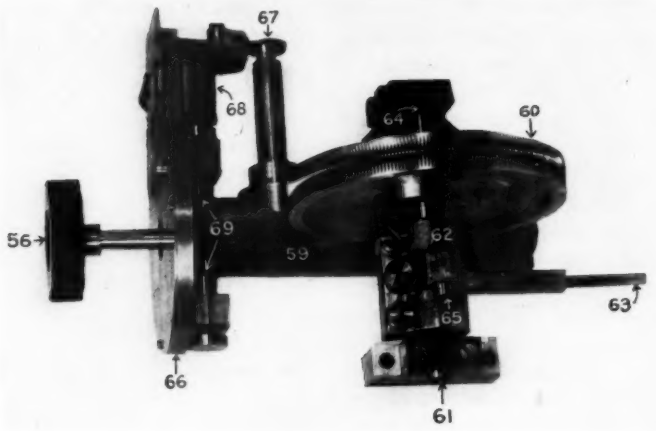

FIG. 9. Fine focusing gear train.

On the best standard microscopes, the slow motion for fine focusing is such that one scale division represents ± 0.002 mm or ± 0.001 mm. Our motion is one hundred times as slow as the latter. It therefore has the disadvantage of requiring excessive time if a considerable amount of travel of the objective is needed to bring the focus to perfection (since to move 1 mm would require 500 full turns of the focusing knob).

In order to retain the advantages of the slow motion and avoid excessive use of time, we provided for drive of the worm by motor. A tiny reversible, variable-speed motor is rubber-mounted inside the base casting of the microscope, and connected to the worm by chain and sprockets, with spring tension that obviates back-lash in the chain. The design of the switch and the amount of resistance in the circuit are such that the motor may be operated at either of two speeds: if the image is evidently far from focus, the rapid motion is used and moves the objective at about 0.01 mm per second; when the image is becoming plain the slower speed is used, which changes at about 0.004 mm per second, or respectively about 25 and 10 times as fast as can be done manually.

Originally, we intended this motor focusing only for the purpose of approximating true focus, and expected that final perfection would have to be attained by the hand-knob. But the response of the motor to the two-way switch, which has a very small throw and the "feel" of a modern high speed telegraph key, permits us to control the focusing by motor as delicately and certainly as can be done by hand, one division at a time, if needed; so manual focusing is now rarely used. As in the case of the coarse focusing, the control is possible from either of two switches, one in convenient reach when the operator is seated before the observation ocular, the other at such point near the back end of the optical bed that focusing can be accomplished at all positions of the bellows-draw. The switches for coarse and for fine control are mounted under the same guard (see Fig. 5), but are of different height and shape and so separated that only one at a time may be moved; therefore, inadvertent operation of the wrong switch is very unlikely. The hand knob is ordinarily out of gear, and therefore is idle when focusing is done by motor; for manual focusing, the hand knob is pushed into clutch, which simultaneously puts the motor out of gear. Conflict between the two is thus prevented. By pressing either switch lever to the left, all the motor-driven motions operate to increase the distance between objective and object; this is the same as if the objective itself were pushed to the left. Conversely, the distance shortens when either switch lever is pressed to the right. By following this mnemonic scheme

and by always moving the objective first *away* from the object (while observing the effect on focus) and only *toward* when it has been established that this is the necessary direction, damage is avoided for objective and object.

Accidental overdrive of the fine focusing mechanism is prevented for manual operation by the automatic and sustained sounding of an electric buzzer as soon as the intended limit of travel in either direction is reached; for motor driven operation immediately after the sounding of the buzzer, the current cuts out for that direction but remains on for the reverse direction. These safety controls have proved dependable.[16]

To one familiar with the conventional focusing mechanism, our exceedingly slow motion might seem utterly unnecessary. But our experience justifies the design adopted; some details are given under the section headed "Performance and Possibilities."

Optics

Aside from the straight-line axis of the image-bearing beam and the provision of generous apertures, as already described, our instrument involves no departures from conventional application of optical principles. Most of our optical parts are standard products of the better American and European makers; only a few parts, such as polarizing prisms, the telecentric correcting lenses for the analyzer, and the quartz wedge, had to be made especially for us, and then only because of our insistence on large diameters.

Limited resources necessitated a choice of either compromise and restriction of the design and quality of our instrument proper, or else temporary contentment with our existing equipment of objectives and oculars assembled during past years. We chose the latter alternative. For we believe that our new instrument now affords the basis for a real advance over our past standards.

[16] Dr. L. V. Judson, Chief of the Length Section, National Bureau of Standards, has kindly brought to our attention his experience with another early example of motor drive of a precision instrument. "The comparator to which I alluded is the 4-Meter Geodetic Comparator of the Russian Government. It is illustrated in Fig. 9 of General Pamphlet No. 405 of Société Génevoise d'Instruments de Physique of Geneva, Switzerland. A description of it is given in Russian with a Resume in French (in volume 5 of Travaux de l'Institut de Metrologie et de Standardisation de l'U.R.S.S.) published in Petrograd and Moscow in 1934. The focusing could be either manual or electrical. In checking up this machine at the factory in Geneva it was observed by myself and others that focusing by the use of the electric motor with its uniform motion appeared to give more consistent results than were obtained with the hand operation."

Performance and Possibilities

After four and a half years of use, the instrument has proved capable of delivering most of the results that were hoped for when its construction was started. Some disappointments have been encountered, as detailed on earlier pages; of most of these the cause has been removed, but a few as yet persist.

Inasmuch as the fine focusing mechanism is the least conventional part of the instrument, both as to intended achievement and as to specific design, a definite record of its performance is desirable. It must be admitted that the report is not one of unmixed perfection; but in our opinion the advantages attained greatly outweigh the difficulties.

First as to the mechanical precision and performance of the fine focus control. At the outset, the behavior seemed all that we had felt could be expected. For example, when a well polished opaque specimen was magnified at 3000–6000 diameters, we found that, after securing the best determinable focus[17] and recording the dial reading, we could throw the image completely out of focus by the fine mechanism, and upon

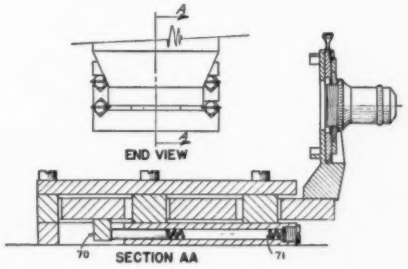

FIG. 10. Longitudinal and cross sections of objective carriage.

[17] Aptly defined by Dr. W. B. Rayton of Bausch and Lomb as "when the eye is satisfied that definition is as good as can be made."

again restoring best visual focus the dial would be found to stand within 3 to 10 scale-divisions of its first reading—and this could be done repeatedly. What this meant was that with an objective of N.A. 1.40 and essentially monochromatic light, we could consistently determine focus within limits of 300 to 1000A, or say from 1/10 to 1/5 of the wave-length of the light being used. This was checked by such careful manipulators as Dr. George Tunell and Dr. C. J. Ksanda of the Geophysical Laboratory of Washington.

With time, however, and before we had opportunity to make the intended interferometer tests, we began to recognize erratic performance, involving both back-lash and lag-and-creep, differing in amount at different places along the total range of travel. When this had increased to quite disconcerting proportions (as contrasted with the previous excellence), we made interferometer tests, using the λ5461 line of the mercury arc, which should give a ratio of 27.305 scale divisions per interference fringe if each scale division were absolutely 100A as our design intended. This disclosed long good stretches (e.g., 11 fringes = 302 divisions; ratio 27.45); but also great variations, with extremes from ratios as low as 9 to as high as 124 at points where the direction of motion was reversed and back-lash and lag entered excessively. The mechanism was therefore disassembled. We found that the porpoise-jaw oil had become somewhat gummy; also that the perforated brass strips used as spreaders for the balls of the objective-carriage had become somewhat corroded by the lubricant and tiny flecks of the verdigris had adhered to balls and ball races, rendering them uneven. New ball-spreaders of stainless steel and a stiffer returning-spring were substituted; the worm, worm-wheel, screw and nut of the gear-train were very carefully relapped and reset to correct snugness; and a new oil of lighter grade and better stability was introduced. The earlier precision was restored. Interferometer measurements were repeated (about a year ago), with results as follows at different parts of the travel, the + readings indicating motion in one direction and the − readings the opposite direction:

+ 7 fringes[18]	= 194 scale divisions;	ratio 27.71
−14	373	26.65
+ 7	191	27.29
− 6	167	27.83
+13	363	27.92
		Theoretical 27.305

Slight back-lash has reappeared, for reasons not yet run down. It is not serious, and we hope will not become so. For the present, at any rate, it causes little difficulty in attaining perfect focus; and as the back-lash in one direction appears identical with that in the opposite direction, the dial readings can be used reliably *if always read when the motion is in the same direction.*

By means of the Zeiss Multiplicator a succession of test strips may be photographed at progressively differing (recorded) focus settings so chosen as surely to include that focus nearest to perfect (see Fig. 11). The focus is then set to the same reading as for the test strip showing the best focus (i.e., 167,360 in Fig. 11) and a full plate then exposed with confidence that its focus will be correct. For special subjects at high power this is a marked convenience. It is also possible by this means to calibrate at various magnifications the focus difference for the several wave-lengths ordinarily used; and then, first focusing visually at high power with green (λ5461) to which the eye is most sensitive and for which the mercury arc is most intense, set off the appropriate scale-difference and expose in the blue or violet. When and if we install optics for ultraviolet, it is believed this scheme of focusing can likewise be depended upon.

As for the future, it seems possible to entertain the idea that the positive and precise control of focus as now made available has further use than mere convenience and the reduction in ratio of spoiled negatives. Our own experience has led us to the impression that high power lenses can be improved from present standards. Lenses of the same specification from a given maker vary in excellence when tested under

[18] The reading for every fringe was recorded; an occasional one showed deviation from the average for its group; the maximum such deviation, in a single instance, reached 9 divisions, and probably should be disregarded as observational error.

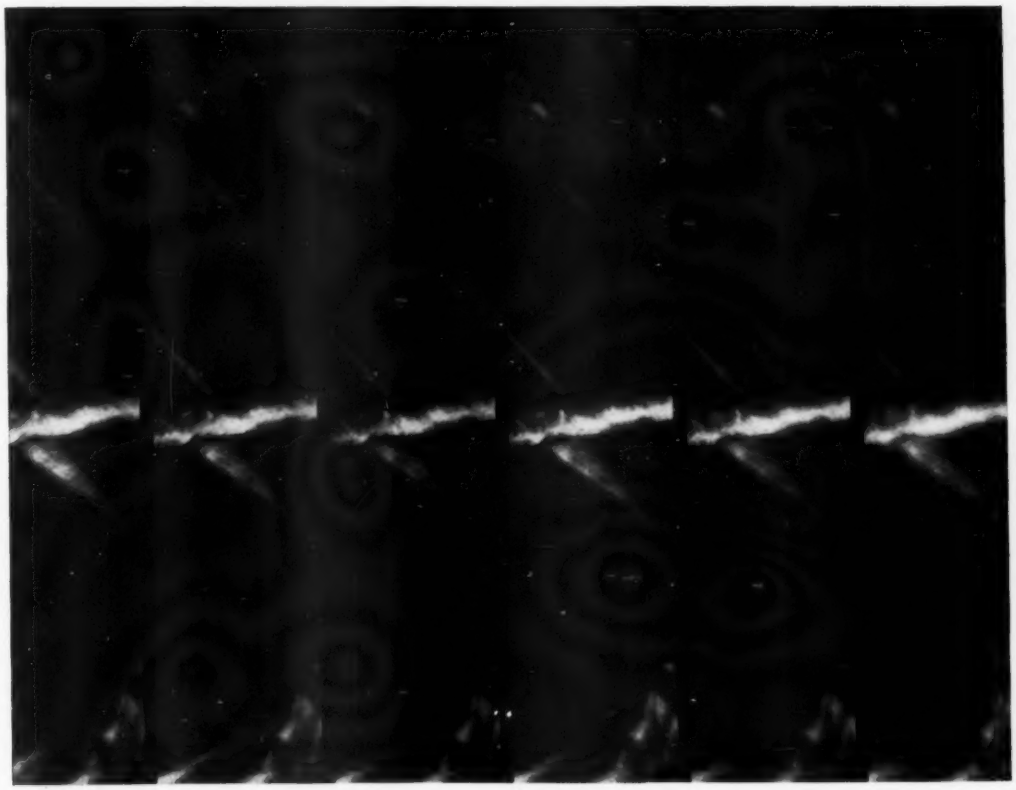

FIG. 11. Reproduction of negative taken to test the perfection of focus and resolution. General field consists of bornite (dark) with alteration lenses of chalcopyrite (bright) which are bordered and partly replaced by covellite (mottled). Black diagonal line in lower part of field is a polishing scratch. Scale division readings of successive focus settings are indicated, the focus interval being 0.1 micron. Best focus lies between '60 and '70, but nearer to '60. Plane polished surface, unetched. Photograph at 3175 magnifications by means of Bausch and Lomb 3 mm, 1.4 N.A. apochromatic objective, short-mounted, 215 mm tube length; "high ampliplan" ocular. Illumination: high pressure capillary mercury arc, filtered to λ 4358. Resolution appears to cease where the wedge-shaped blade of chalcopyrite has a width of 1070A.

rigorous focus control. This seems inevitable when one considers how tiny are the all-important front lenses and thus that the slightest deviation from the computed curvature or local infinitesimal deviation from the average refractive index would impair the quality. But it seems hardly to be doubted that if the makers themselves had means to control the focus precisely, their routine tests would lead to selection for sale of a higher average quality than at present. If the proportion of rejects resulting from these more exacting standards should be excessive, the science would win; for almost certainly the lens makers' would be stimulated to a new level of effort.

That such effort would not be futile seems highly probable. The lens makers have for decades worked under the complacent conviction that maximum theoretical resolution for 1.40 N.A. objectives had been actually reached—at around 1500 magnifications. Lucas appears to have been the first to cast doubt on the validity of the assumptions on which the Abbé formula for resolving power is based. And his recent remarkable results[19] with the 1.60 N.A. objective

[19] J. Frank. Inst. **217,** 705–6 (1934).

and mono-chlor-naphthalene as immersion fluid, showing increased resolution up to 4000 diameters, further indicate the desirability of re-examining the classical conception of the limit of useful magnification.

So long as the makers accepted the conventional limit as valid and had already attained it, there was little incentive toward progress. But with that limit apparently surpassed, there is no present knowledge as to how far ahead the true limit may lie. If present-day objectives do substantially better than the "limit" for which they were designed, is it not reasonable to suppose that effort to do better still may conceivably be rewarded?

If it be permissible for practical microscopists to make suggestions for the consideration of the experts in optical theory and design, we might mention two points:

First: That computation should never be relied upon solely—no matter how venerable the underlying theory—if practical check by actual measurement can possibly be made. For example, in the informal discussion of our paper at the Philadelphia meeting, our report that difference in perfection of focus can be detected visually when the objective : object separation is changed by as little as 300–1000A was met by the disconcerting statement that the (*computed*) depth of focus renders such result unattainable. It is now possible to submit evidence on this point for examination. The successive strips of Fig. 11 were taken at focuses differing progressively by 10 scale divisions or 1000A. These strips plainly show the passage from one side to the other side of perfect focus, which appears to lie close to the reading 167,360, since the contiguous '50 and '70 strips are obviously less perfect; but '70 is rather better than '50, therefore perfect focus in this instance probably lies in the neighborhood of 167,363. At '40 or '80 the focus is definitely bad; *but this might not surely be recognized without the immediately adjacent better focus for comparison.* We leave for those more expert in such matters final decision as to whether "depth of focus" needs review; but it seems permissible to suggest that the decision be made in the light of what it is now possible to do. The same applies to limit of resolving power, considered later.

Second: That insofar as possible the various tests of lens quality be made upon object materials best suited for the purpose. The existing literature conveys the impression (which one's own range of information tends to confirm) that microscopical theory and testing rest very largely on application to objects of finite thickness for examination in transmitted light. These preparations range from a few microns to some tens of microns thick; their marginal edges vary through all angles away from the optical axis of the lens; their structural and chromatic features overlap and confuse one another in different planes; they generally require mounting glass and cover glass, with intervening immersion fluid. Plainly, such preparations are far from ideal as a basis for microscopical theory or the rigorous testing of high power lenses. In such a miscellaneous pile of material, therefore, it is hardly surprising that uncertainty should have existed regarding the exact plane of perfect focus and that questionable conclusions as to depth of focus should have been drawn. Nor is it strange that by the time 1000 or 1500 magnifications should have been reached on this confused three-dimensional array, further magnification would yield nothing useful.

It is probably not without significance that the great majority of satisfactory and effective microphotographs taken at magnifications substantially exceeding what the Abbé formula would indicate to be useful have been of polished sections of opaque material by reflected light. Here the object approximates very much more closely to an optical plane, and the other complications and disadvantages of thin sections are absent. Such metallurgical photography as that by Lucas and Vilella at magnifications of 3000 to 4000 sets a standard which thin-slice photography can hardly hope to touch.

Nevertheless, the structure revealed in most metallurgical microphotographs, and especially those at very high powers, is due not nearly so much to different actinic value as it is to a *difference in level* of the respective components, occasioned by differential etching or else by hardness differences brought out by polishing. As a result, each unit of the revealed structure is bordered by a band inclined to the general plane of the section—a band representing the

transition from the high level to the low level of the adjacent components. These bands, reflecting the incident light obliquely away from the optical axis, photograph very dark or black. *It is the presence of these "shadow borders" that largely constitutes the picture;* if they were eliminated, little more than a uniform field would remain. Since these shadow borders have finite width (which depends on the relative character and attitude of the adjacent components and on the conditions of etch or polish) they give relatively broad images on the negative. *And additional enlargement merely magnifies their breadth, without revealing any more detail* (except insofar as higher N.A. may catch somewhat more of the obliquely reflected light and thus record whatever image this may contain). That this is true as to character of the surface and cause of the picture is readily confirmed by examination in obliquely incident light, or so-called "reflected dark-field," where these inclined bands show bright; and because the incident beam is of relatively small aperture they appear narrower than before, besides sometimes showing detail since now they reflect into the objective.

Thus, metallurgical preparations, although vastly more satisfactory than transparent slices as test material, are rarely if ever ideal; and the present limitations to useful magnification are ascribable *not so much to deficiencies of the lens as to unsuitable geometry of the surface photographed.*

In mineralography, which deals with opaque natural minerals, it has become conventional to take microphotographs of the polished surfaces without etching, because so many of the mineral species possess sufficiently unlike chromatic and reflecting characteristics to affect the photographic plate differentially. Because great differences in hardness among these minerals produce troublesome "relief shadows" even at low powers when polishing is done by the conventional fabric-covered laps, a method of polishing has been developed by soft lead laps,[20] which gives a mirror-smooth and almost optically plane[21] surface even on minerals of widely differing hardness. When a specimen of differently colored minerals polished in the *same plane* is photographed, the picture is produced *chromatically;* boundaries of adjacent components are not concealed under inclined planes of finite breadth but are revealed as clear-cut *lines* of separation.

Figure 11 is of such a preparation; it consists of alteration lenses of chalcopyrite, bordered and intergrown with covellite, controlled by crystallographic directions in the general field of bornite. To the metallographer this microphotograph may at first seem flat and disappointing. But this is because there are no strikingly contrasty black "shadow borders" to give it the appearance of crispness; on the contrary the actual contacts between the components are clearly shown, instead of being blotted out. This particular subject was chosen for present purposes because the blade of chalcopyrite running diagonally downward to the right near the center of the field tapers down to very narrow width. There is, indeed, independent evidence (which need not be argued here) for the conclusion that this and similar blades actually taper, at approximately the same angle, to zero width. It would appear, therefore, that this is an excellent subject on which to test resolving power; since if magnification were infinite and resolution perfect the image should be an ideal wedge.

On the negative from which Fig. 11 was printed, the blade mentioned narrows down to a width, measured by filar micrometer, of 0.34 mm while just beyond this it appears to round off and terminate abruptly; this appears to be the place at which resolution breaks down. As the magnification of the negative is 3175, this narrowing blade of chalcopyrite has an indicated true width, where resolution ceases, of about 1070A, or less than one-quarter of the wave-length 4358A used in photographing it. Since effects of grain-size of negative, halation, and possible vibration during photography would all apparently tend in the direction of exaggerating the true size of the image on the negative, it would

[20] J. W. Vanderwilt, Econ. Geol. **23**, 292–316 (1928); also Graton, Am. Mineralogist **22**, 492–5 (1937). As a matter of fact, since by far the larger part of our high power work deals with polished sections of ores, the elaborate and expensive microcamera here described would not have been justified at all, either for Harvard or for Ottawa, if a method of polishing had not been available to give almost ideally plane surfaces for examination.

[21] Maximum level difference between hard pyrite and soft gold when so polished has been repeatedly measured as less than 0.25 micron; many pairs of minerals have no discoverable difference of level on the polished surface—certainly less than 0.01 micron.

seem safe to conclude that the lens combination used in this case has a resolving power better than $\lambda/4$. The diagonal dark scratch shown in Fig. 11 is narrower still; indeed, we have on other occasions photographed polishing scratches which we believe have a width of the order of $\lambda/10$.

Just as in the matter of depth of focus, it must be left to those more competent to decide whether the results that can now be secured from best preparations and best focus necessitate review of the theory of microscopical resolution. But even if it should prove that the present formulae are correct, it seems to us that high power lens making is today being conducted in an empirical range, in the sense that actual optimum performance appears better than previously supposed.

May we close with the plea that the lens makers, without waiting for a better theory—if one be needed—make effort to surpass their present best. If they can do better, we believe our instrument will recognize and record the improvement. We need the best high power objective for vertical illumination that can be made. If advance from present quality of such a lens can be achieved, there should be many who would join us in securing them. But to test them and to obtain invariably their *best* results we believe they must be handled under more precise control of focus than has hitherto been available.

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Observations of a Searchlight Beam to an Altitude of 28 Kilometers

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The beam of a high intensity searchlight directed over an observing station 18.4 km distant at angles of 30° and 45° to the horizontal was visible to a vertical altitude of about 20 km and was photographed to 28 km on clear nights. The intensity of the beam measured from the photographs was of the same order of magnitude as that calculated from the Rayleigh theory of molecular scattering using standard tables of stratospheric densities. At 5 km the observed intensity was greater than the theoretical intensity by a factor of about 7, the factor decreasing to about unity above 10 km, as would be accounted for by a small number of haze particles at 5 km which decreased to an imperceptible amount above 10 km.

INTRODUCTION

THE use of a searchlight to investigate the atmosphere to considerable altitudes has often been proposed. Synge[1] suggested the concentration of several searchlight beams at a high region and the measurement with a photoelectric cell of the light scattered from the region. With a sufficient number of searchlights, it appeared that the light scattered from an altitude as high as 30 km might be detected, and that the limit of altitude obtainable would finally be set by the background illumination of the night sky and the stars. To reduce the limitation Tuve, Johnson and Wulf[2] discussed the possibility of employing a modulated beam and detecting by a photoelectric cell and amplifier tuned to the modulation frequency. From the known characteristics of large searchlights and of the amplification of photoelectric cell circuits they estimated that the light scattered from altitudes above 50 km might be measurable. The only recorded experimental observations appear to be those of Duclaux[3] who photographed from a distance of 2.4 km the vertical beam of a 5 ampere arc at the focus of a 30 cm mirror. With an $f:1.5$ lens and an exposure of 1 hour and 30 minutes on a clear moonless night and the beam was traced on the plate to an altitude of 3.4 km. It was of interest to carry the experiment further, and as described in the following paragraphs the beam of a high power searchlight was photographed in clear weather to an altitude of about 28 km. The measured intensity of the beam was in fair agreement with that attributable to molecular scattering, being slightly in excess as would be accounted for by a few haze particles.

[1] Synge, Phil. Mag. 9, 1014 (1930).
[2] Tuve, Johnson and Wulf, *Terr. Mag. and Atmos. Elec.* (1935).
[3] Duclaux, J. de phys. et rad. 7, 361 (1936).

EXPERIMENTAL OBSERVATIONS

A high intensity carbon arc searchlight was provided and operated by the United States Army Engineers. The mirror was 152 cm (60 inches) in diameter and 62 cm in focal length, the arc consumed 150 amperes at 78 volts, or 11.7 kilowatts. The angular width of the beam was 1.2°. Observations were made from a station in Virginia 18.4 km to the northwest of the searchlight; there was no disturbing artificial illumination. A camera with an $f:1.8$ lens and 45×60 mm plates was used to photograph the beam. The night of April 12, 1937, was clear with no moon; the beam was directed over the observing station at an angle of 30° to the horizontal. Four pictures were taken which are shown in Fig. 1, the vertical altitudes of the beam and the times of exposure, derived from the star tracks, being marked alongside of each picture. Pictures B, C and D were taken on hypersensitive panchromatic plates and A on Verichrome film. The geometry of the scene and the portion of the beam on each plate is drawn in Fig. 2 in which S and P were the positions of the searchlight and the observers, respectively. Up to an altitude of 1.8 km the beam was bright and unpolarized indicating scattering by haze. At this point there was a sharp step-down in intensity, seen in A, Fig. 1, as the beam passed out of the stratum of low lying haze. The haze was actually very slight, the

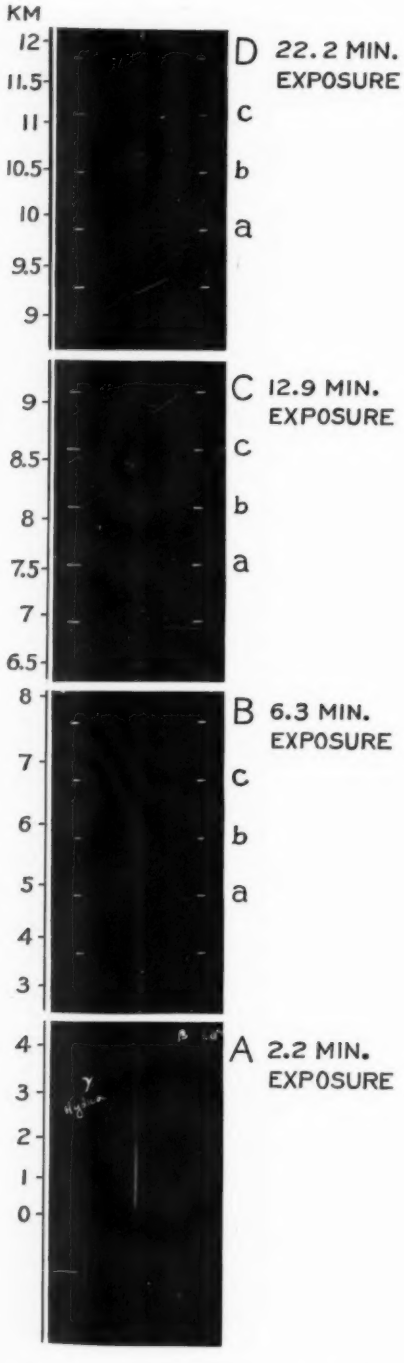

FIG. 1. Photographs of searchlight beam.

atmosphere near the ground being normally, but not abnormally, clear. The increased intensity of the beam in the stratum of faint haze was entirely expected, for it is a familiar fact that the lateral scattering of a beam of light is enormously increased above that due to air molecules by what would be ordinarily termed an almost unnoticeable amount of haze. Above 1.8 km the brightness of the beam decreased slowly and uniformly along the beam until it faded to indistinguishability against the stars. It was visible to the eye to more than 30° beyond the zenith or to an altitude above 18 km. Tests with yellow and orange filters showed that for altitudes above 5 km a considerable portion of the light of the beam was the blue part of the spectrum. To the dark adapted eye the beam appeared of course whitish rather than bluish.

In Fig. 1, *D* the beam was easily visible entirely across the plate amid the star tracks and was considerably brighter than the night sky background. The short horizontal lines at *a*, *b* and *c* alongside of *B*, *C* and *D*, Fig. 1, mark the sections of the beam across which densitometer records were made. The negatives were less exposed at each end than in the central portions because the camera lens yielded its full aperture only in the central half of the negative. Therefore only the sections *a*, *b* and *c* in the central portion of each negative were used for intensity measurements.

In another experiment on May 10, 1937, the beam was pointed up at an angle of 45° to the horizontal passing through the zenith of the observing station. The night was very clear and without a moon; there was no noticeable concentration of slight haze near the ground as occurred on April 12. The beam was visible to a point slightly beyond the zenith or to an altitude of about 20 km. With the camera directed overhead to embrace the region *E*, Fig. 2, a single photograph was taken with an exposure of 54 minutes between 9 and 10 P.M. The region *E* was purposely chosen to yield the photographic limit of the beam under the experimental conditions which prevailed. On the negative the beam could be followed to 28 km but not to 30 km. The negative was not suitable for reproduction for the trace of the beam was too faint; the photographic limit of the beam will always be too faint for

reproduction. There was a general fogging due to night sky light and stars, and a longer exposure would not have given a greatly increased length of the beam. During the early part of the exposure the zodiacal light was visible in the west and an aurora in the north; these apparitions were faint, but were unusually strong for this latitude. The general night sky illumination may have been unusually strong also, although it was not perceptible to the untrained eye. It seemed probable that the altitude of 28 km was not far from the limit attainable by photography of the 11.7 kw searchlight. To reach higher altitudes a more intense beam would be necessary and a more favorable climatic location as a mountain top would be desirable.

The altitude 28 km was about 6 km above the height attained by the stratosphere balloon[4] Explorer II. It was probably well into the ozone region, the lower boundary of which was at about 20 km according to the observations of Explorer II.

The atmospheric conditions during the above observations were the clearest which occurred during the experimental program. When the atmosphere was not clear the haze affected the beam as anticipated. For example, a fair amount of haze brightened the beam very much at low altitudes by increased scattering and decreased the brightness at the higher altitudes by absorption, the decrease being due to the degradation of the beam in its passage through the haze and to the fact that the observers on the ground were looking at the beam through haze.

Observed and Theoretical Intensity of the Beam

It was of interest to compare the intensity of the light of the beam determined from measurements of the photographs with the intensity calculated from the scattering of the air molecules. The complete details of the two sets of calculations are given in Appendices 1 and 2, the more important steps are mentioned briefly here.

In order to measure the intensity of the beam from the negatives a part of each negative had previously been cut off and kept in reserve. A calibrated tungsten lamp with a blue filter served as a source of known intensity of light of spectral characteristics approximately the same as those of the scattered light of the beam. With the source at various distances a series of spots of known intensities were impressed on the reserved pieces of each negative, the intensities being so chosen that the weakest was weaker than the beam and the strongest was stronger than the beam. All the negatives and the reserved pieces were developed in the same developing bath at the same time. Densitometer records were taken across each negative at sections *a*, *b* and *c*, Fig. 1, and also across each spot on the reserved pieces. From these, omitting tedious and obvious details, the intensities of the central portion of the beam at various altitudes were determined and are given in column (3), Table I.

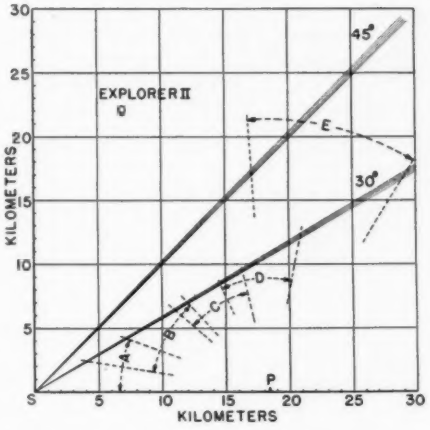

FIG. 2. Geometry of searchlight scene.

It may be noted in passing that the energy flux from a sheet of white paper in the light of the full moon is about 1 erg cm^{-2} $sec.^{-1}$. This is about 10 times the intensity of the beam at an altitude of 4.8 km, shown by *a*, negative *B*, Fig. 1.

The calculation of the intensity of the beam from the theory of molecular scattering made use of the known candlepower of the beam and the spectral intensity distribution of the light of the high intensity carbon arc. The flux of energy in each wave-length was calculated at various distances from the searchlight on the assumption that absorption of the beam by the atmosphere was relatively small. The molecular densities at the various heights were taken to be those of

[4] National Geographic Society, U. S. Army Air Corps Stratosphere Flight of 1935.

TABLE I. *Intensity of searchlight beam.*

(1) Negative	(2) Altitude	(3) Intensity of Beam erg cm^{-2} sec.$^{-1}$ From Photograph	(4) Intensity of Beam erg cm^{-2} sec.$^{-1}$ From Theory	(5) Ratio (3)/(4)
E	22 km	$(0.7)\times10^{-5}$	0.49×10^{-5}	(1)
D *c*	11.0	0.54×10^{-3}	0.22×10^{-3}	2.5
b	10.4	0.54×10^{-3}	0.24×10^{-3}	2.3
a	9.8	0.54×10^{-3}	0.25×10^{-3}	2.2
C *c*	8.7	0.93×10^{-3}	0.28×10^{-3}	3.3
b	8.2	1.9×10^{-3}	0.34×10^{-3}	5.6
a	7.6	2.5×10^{-3}	0.39×10^{-3}	6.4
B *c*	6.7	3.8×10^{-3}	0.59×10^{-3}	6.4
b	5.8	6.3×10^{-3}	0.94×10^{-3}	6.7
a	4.8	9.7×10^{-3}	1.34×10^{-3}	7.2

Table II, which were derived from Humphrey's tabulations.[5] The values were changed by less than 5 percent by the use of the actual atmospheric pressure and temperatures observed during airplane flights to 5 km on the days of the searchlight experiments. Introducing the above quantities into the Rayleigh scattering formula, and integrating across the beam at the required angles, yielded the theoretical values of the intensity of the beam corresponding to those determined from the photographs. They are given in column (4), Table I.

It is difficult to estimate the possible errors in the numbers of columns (3) and (4), Table I; they, and those of column (5), may be correct within a factor of 2. With this qualification the theoretical intensities of column (4) deviate in a regular manner from the observed intensities of column (3) as shown by the ratios of column (5). The ratios decrease from about 7 at 5 km to unity at 10 or 15 km. The deviations would be accounted for by the assumption of a small number of scattering particles, such as haze, which decreased with increasing altitude. About 100 particles cm^{-3} of diameter 0.1μ and albedo 1 would yield the factor 7; or, a larger number of smaller particles of less albedo. Such a haze distribution would increase the intensity of the beam in the lower levels above that due to molecular scattering without at the same time attenuating the beam appreciably in the higher levels.

It is concluded that for a relatively clear atmosphere in levels above 5 km the intensity of the searchlight beam was of the order of magnitude of that calculated from the theory of molecular scattering using the density of air molecules of a standard atmosphere, Table II. At 5 km the observed intensity was greater than the theoretical intensity by a factor of 7, the factor decreasing to about unity above 10 km, indicating the presence of a small number of haze particles at 5 km which decreased to an imperceptible amount above 10 km.

In conclusion it is a pleasure to acknowledge the cooperation of Major William Sackville, U.S.A., in arranging for the operation of the searchlight.

[5] Humphreys, *Physics of the Air* (1929), p. 74.

APPENDIX 1

Photographic determination of the intensity of the beam

A ribbon filament tungsten lamp, standardized by the National Bureau of Standards, burning at a red (λ0.655μ) black body apparent temperature of 1719°K, served as a known source of energy. A diaphragm limited the exposed area of the filament to 0.124 cm^2. A selected blue filter of known transmission was placed in front of the lamp; the spectral energy curve of the light through the filter is given in curve 1, Fig. 3. The calculated spectral energy curve of the scattered light of the searchlight beam is

TABLE II. *Average temperature and density of the atmosphere.*

Altitude	Temp.	Density d	n	Altitude	Temp.	Density d	n
0 km	1.72°C	1256×10^{-6}	261×10^{17}	20 km	−55°C	89.7×10^{-6}	18.6×10^{17}
2	−4.16	1010	210	22	−55	65.8	13.7
4	−15.3	817	170	24	−55	48.2	10.0
6	−29.3	660	137	26	−55	35.3	7.32
8	−43.6	529	110	28	−55	25.9	5.39
10	−54.2	414	86.0	30	−55	19.0	3.95
12	−55	311	64.5	32	−55	13.9	2.89
14	−55	228	47.5	34	−55	10.2	2.12
16	−55	167	34.7	36	−55	7.54	1.57
18	−55	121	25.2	38	−55	5.53	1.15
				40	−55	4.07	.84

given in curve 4, Fig. 3. The curve is limited to the wave-length region from about 3800A to 7000A, the lower limit being set by the transmission of the camera lens and the upper limit by the sensitivity curve of the photographic plates. Curves 1 and 4 cover approximately the same spectral region, and since the sensitivity of the photographic plate did not vary rapidly through the region, the tungsten lamp with the blue filter was considered a satisfactory source for calibration purposes. The total energy emitted in a hemisphere by the filament through the filter, given by the area under curve 1, Fig. 3, was 2.58×10^3 erg sec.$^{-1}$, and the flux at a distance r was

$$2.58\times10^3/2\pi r^2 \text{ erg cm}^{-2} \text{ sec.}^{-1}. \quad (1)$$

With the lamp at six different distances six spots were photographed on the reserved pieces of the negative with 5 minute exposures, each spot being caused by a known intensity of light determined by (1). As already mentioned, densitometer traces of the spots were made from which the intensities of the central portions of the beam at various altitudes were determined, the values being given in column (3), Table I.

Appendix 2

Theoretical calculation of the intensity of the beam

The spectral intensity curve of the high intensity carbon arc from the measurements of Kalb[6] is shown in curve 2, Fig. 3, in which the intensity i_λ is plotted as ordinate against the wave-length λ as abscissa. The observations extended only to 1.2μ; the curve was extrapolated to longer wave-lengths as indicated by the dotted portion. The hump at 0.38μ is due to the cyanogen band emission. Curve 3 is the spectral energy curve of black body radiation at 5600°K. The agreement between curves 2 and 3 throughout the visible spectrum from 0.4 to 0.7 shows that the spectral characteristics of the visible light of the high intensity carbon arc were similar to those of a black body at 5600°K.

At 1000 yards from the searchlight the luminous intensity was 28, 24, 14 and 4 lumens ft.$^{-2}$ at 0°, 0.2°, 0.4°, and 0.6°, respectively, from the center of the beam. Therefore the average brightness of the beam across 1.2° was 18 lumens ft.$^{-2}$ or 1.93×10^{-2} lumens cm^{-2}. The intensity of the searchlight did not fluctuate more than 5 percent during an hour's operation. Multiplying 1.93×10^{-2} by 1.5×10^{-3} to convert to luminous watts,[7] by 1/0.132 to convert to mechanical watts and by 10^7 to convert to ergs, yielded 2.19×10^3 erg cm^{-2} sec.$^{-1}$. From this the energy flux i of the beam at any distance from the searchlight was calculated from the inverse square law. The number 0.132 is the radiant luminous efficiency of black-body radiation at 5600°K.[8]

Further,
$$i=\int_0^\infty i_\lambda d\lambda, \quad (2)$$

and is given by the area under curve 2, Fig. 3.

[6] Kalb, Elec. Eng. **56**, 319 (1937).
[7] Hyde, Forsythe and Cady, Phys. Rev. **13**, 45 (1919).
[8] Coblentz and Emerson, Bul. Nat. Bur. Stand. **14**, 255 (1917).

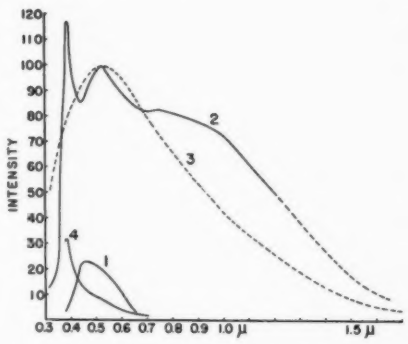

FIG. 3. Spectral energy curves, 1 tungsten lamp through blue filter, 2 high intensity carbon arc, 3 black body at 5600°K, 4 curve 2 scattered by air.

If a beam of unpolarized light of intensity i_λ erg cm^{-2} sec.$^{-1}$ and wave-length λ passes through a gas of refractive index μ and n molecules cm^{-3}, the intensity $i_{s\lambda}$ of the light scattered at an angle θ to the beam is

$$i_{s\lambda}=i_\lambda 2\pi^2(\mu-1)^2(1+\cos^2\theta)/n\lambda^4. \quad (3)$$

(3) is derived from the Rayleigh[9] theory of scattering by particles small with respect to λ. The $i_{s\lambda}$, λ curve for the radiation of curve 2 scattered by air is plotted in curve 4, Fig. 3. Integrating (3) over a sphere gives, as it should, the Rayleigh expression for β the attenuation due to scattering,

$$\beta=32\pi^3(\mu-1)^2/3n\lambda^4.$$

Now
$$\mu-1=\alpha n, \quad (4)$$

where α is a function of λ. α has been measured[10] for air throughout the wave-length region from 2000 to 10,000A. Substituting (4) into (3) yields

$$i_{s\lambda}=i_\lambda 2\pi^2 n\alpha^2(1+\cos^2\theta)/\lambda^4. \quad (5)$$

(5) expresses the scattered intensity explicitly in terms of the molecular density n.

The total scattered light i_s from unit volume of the beam in wave-lengths from 3800 to 7000A is

$$i_s=\int_{3800}^{7000} i_{s\lambda}d\lambda, \quad (6)$$

and is given by the area under curve 4, Fig. 3.

The intensity of the beam i_b is the total scattered light from the beam in a direction θ, and is the integral of i_s across the beam at angle θ. Since the beam diverges only slightly we have to a close approximation

$$i_b=i_s f/\sin\theta, \quad (7)$$

where f is the diameter of the beam at the point in question.

By means of Eqs. (2) to (7) and n from Table II, the values of i_b were calculated for a, b, and c of pictures B,

[9] Rayleigh, Phil. Mag. **51**, 107, 274 (1871); **47**, 375 (1899).
[10] Meggers and Peters, Bul. Nat. Bur. Stand. **14**, 731 (1918).

C and *D*, Fig. 1, and for an altitude of 22 km from picture *E*. They are tabulated in column (4), Table I.

Appendix 3

Atmospheric molecular density

To determine the molecular density n we make use of the average values of the atmospheric density d and temperature to 40 km tabulated by Humphreys;[5] they are given in Table II, d being in grams cm^{-3}. The values were based on European observations of the temperatures and pressures to 20 km and extrapolation above on the assumptions that the temperature was −55°C from 20 to 40 km and that the atmospheric composition was the same as at sea level. The values were, within 2 percent, the same as those adopted to 24 km as the "U.S. standard atmosphere." [11]

The mass of the average air molecule is 4.82×10^{-23} grams, and

$$n=d/4.82\times10^{-23}. \tag{8}$$

The values of n from (8) are in the last column of Table II.

[11] Nat. Adv. Com. Aer. Technical Report No. 538.

A Case of Dispersion Without Deviation of the Mean Ray

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(Received August 19, 1937)

This paper shows that a direct vision spectroscope may use a glass prism immersed in a liquid contained in a cell with parallel ends. The effect of changing the prism angle is shown, and the data for a 14° dispersion are given.

THE use of Christiansen filters in various ways suggested a method of obtaining dispersion "without deviation." The procedure is quite obvious but the only references[1] which have been found involve somewhat different details, and actually antedate the discovery of the Christiansen effect. Whether the necessity of a suitable temperature control or some other reason has caused the general idea to be laid aside, it still may interest those who wish to use direct vision methods to consider the details here presented.

If an ordinary glass prism is placed in a rectangular cell with plane parallel glass sides, and then covered with a liquid for which the index of refraction is the same as that of the prism for some frequency in the middle of the visible spectrum, light of that frequency will go through the cell without deviation, while light of higher frequencies will be bent one way and light of lower frequencies will be bent the other. By setting the cell with its ends at right angles to the collimated incident light, the refraction to be considered takes place at three surfaces, the two surfaces of the prism and the exit surface of the cell.

Suppose that the dispersion curves for the solid used for the prism and for the liquid are as represented in Fig. 1, then it is clear on referring to Fig. 2 that light of wave-length λ_0 passes directly through cell and prism without change in direction. The angle of deviation produced by the prism for light of any other wave-length may be easily calculated. To simplify the geometry assume that the direction of incidence is parallel to the base of the prism, then the angle δ_1 between the direct ray and the ray emerging from the prism, can be found from the easily derived equations

$$\sin e = \sin A\left(\frac{n_S^2}{n_L^2} - \sin^2\frac{A}{2}\right)^{\frac{1}{2}} - \sin\frac{A}{2}\cdot\cos A \quad (1)$$

and

$$\delta_1 = A/2 - e,$$

where n_S is the index of refraction of the solid, n_L that of the liquid, A the angle of the prism, and e the angle of emergence from the second face of the prism. Then δ_2, the angle after emerging from the cell, comes from the obvious relation

$$\frac{\sin\delta_2}{\sin\delta_1} = n_L.$$

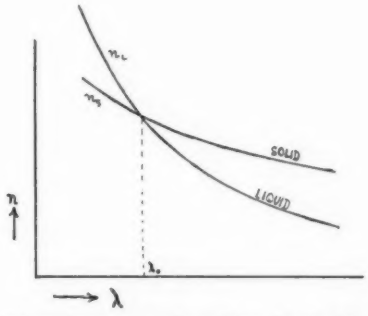

FIG. 1. Dispersion curves of a solid and a liquid of nearly the same index of refraction.

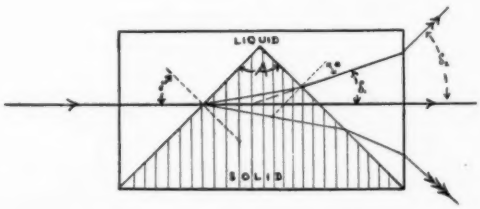

FIG. 2. Horizontal cross section of cell with prism.

[1] Wernicke, Zeits. f. Inst. I, 353 (1881). Zenger, ibid. p. 263.

To check the calculated values, the 90° angle of a prism, No. 1, was used, the prism having an index suitable for use with carbon disulphide. The curves are plotted in Fig. 3, and the dispersions are shown in Fig. 4. It is perhaps unfortunate that the summer temperature brought λ_0 into the violet region, but the agreement between observed and calculated deviations is nevertheless satisfactory. It will of course be noted that in the plotted curves the wavelength for which δ_2 is zero on the calculated curve does not quite agree with that for which δ_2 is zero on the observed curve. This difference is due to two causes, first the experimental work was carried out without a complete temperature control, and second the data for the dispersion of CS_2 were taken from the I. C. tables rather than from the CS_2 actually used.

For the sake of comparison the calculated values of δ_1 have been plotted on the same sheet with the values of δ_2, thus indicating the contribution made by the third surface involved. Further comparison may be assisted by recalling that with a 60° prism of CS_2 the dispersion is about 6°, and with one of a good flint glass it is about 4°.

The fundamental Eq. (1) shows that the angle of the prism is an important factor, and since there is a much greater range of choice in A than in the relative index of refraction, n_S/n_L, it is desirable to learn just how the angle δ_1 varies with the angle of the prism. This was done by calculating δ_1 for a suitable range of values of n_S/n_L for a number of prism angles. These results are shown graphically in Fig. 5 for a short range of values of n_S/n_L, actually a larger range than is generally needed.

From these curves it is clear that the advantage in using a large angle is considerable, and until critical angles of incidence are approached, the main disadvantage seems to be the more or less necessary reduction of prism aperture as the prism angle is increased. The experimental check on this part of the problem had to be confined to prisms already in the laboratory, and there were none of either 120° or 135°, nor were there two of exactly the same glass. Consequently the dispersion curve was observed for a 135° prism made by combining the 90° angle of the prism used in the other check experiment, No. 1, with the 45° angle of another prism, No. 2, of very nearly the same index. The curve, Fig. 6, is shown to indicate the dispersion obtained. If the second prism had been of the same glass as the first the dispersion would have been slightly larger.

As indicated earlier, the successful operation of such a prism combination is dependent upon temperature control, precisely as has been noted by those who have used Christiansen filters. In the experiments reported here, only a rough temperature control was used, and the effects of temperature and temperature gradients were reduced by stopping down the aperture of the beam through the prism. Under these conditions it was comparatively simple to secure definition ample for the purpose.

Perhaps two other features of this procedure should be pointed out. With apparatus arranged as described, the loss of light by reflection from

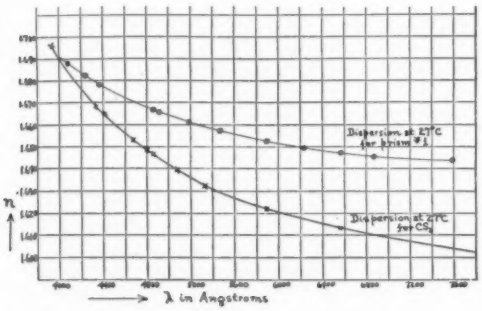

FIG. 3. Dispersion curves for the materials used in one of the experiments.

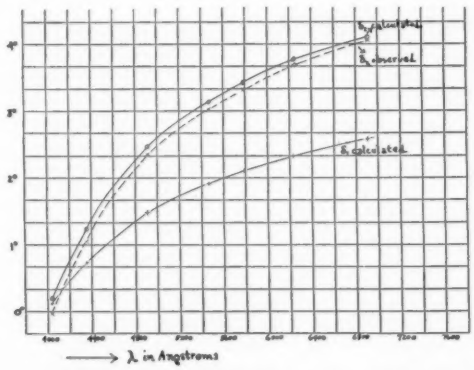

FIG. 4. Deviations for prism No. 1 in CS_2.

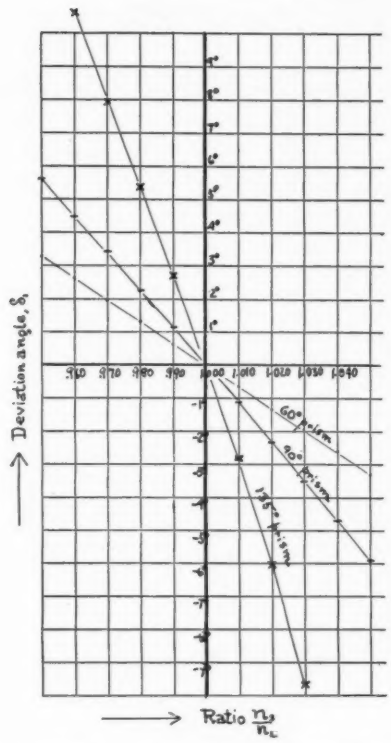

FIG. 5. Dependence of deviation angle on prism angle.

surfaces is reduced to a minimum, e.g. the angle of incidence is nearly zero when the effective index of refraction is greatest. Then the dispersion curve may be made to differ considerably from that of either the glass or the liquid. For example, it can be seen that $d\delta/d\lambda$ in the last curve is much greater in the red than is the case with prisms as ordinarily used.

Dr. W. H. Barkas points out that the equations given above can be combined and simplified, provided that the prism angle and the indices

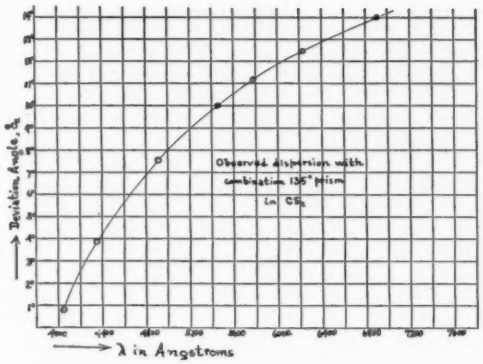

FIG. 6. Observed dispersion with a combination 135° prism in CS_2.

of refraction are such as to justify neglecting terms containing the higher powers of ϵ, where ϵ is substituted for n_S^2/n_L^2-1. If this procedure is warranted the equations give this simple relation

$$\sin \delta_2 = 2(n_S - n_L) \tan A/2.$$

This suggests a comparatively simple way of using the general method for obtaining the dispersion curve for a liquid. For example suppose the prism angle is 90° and its dispersion curve accurately known, then by observing the angles δ_2 for a suitable number of spectral lines the dispersion for the liquid can be quickly obtained from

$$n_S - n_L = \tfrac{1}{2} \sin \delta_2.$$

With the 90° prism, No. 1, which was used in the experiments described the maximum error in computing $\sin \delta_2$ by the approximate equation is about 2 percent, and while it would be larger with increasing angle of prism, there may be cases in which the simplicity of the shorter equation would make it useful.

The Integrating Power of the Eye for Short Flashes of Light

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(Received July 31, 1937)

The integrating power of the eye has been tested for short flashes of light, ranging in duration from 10^{-2} to 8×10^{-9} sec. The shorter flashes were produced by passing the image of the straight filament of an electric lamp across a narrow slit, the light having been reflected from a mirror mounted on an air-driven turbine. The longer flashes were produced by means of a sectored disk. In all cases the number of flashes received by the eye was great enought to avoid flicker and the intensity was well above that required to produce the sensation of vision. It was concluded that the response of the eye depends only upon the total amount of light in the beam and is independent of the length of the light flash. The limit of error was 1.5 percent.

INTRODUCTION

IN 1834[1] Fox Talbot discovered that when a sectored disk is rotated in front of a light source, the response of the eye is the same as if the same amount of light reached the eye from a continuous source. Although questioned for some time, this law was verified by E. P. Hyde[2] in 1906 to within 0.3 percent where the opening of the disk was not less than ten degrees. Because of the use made of the eye to observe very short flashes of light in experiments involving scintillations, Kerr cells, rotating mirrors, etc., it is necessary to extend the observations of Hyde to very much shorter light flashes. In 1929 Chariton and Lea[3] showed that for single flashes in the range from 10^{-5} to 10^{-2} sec. the energy in the flash alone is important and not the duration. In 1934 Beams[4] investigated the response of the eye to still shorter light flashes and concluded that over the range studied (10^{-6} to 10^{-8} sec.) the response of the eye depended only upon the integrated amount of light in the beam and was independent of the length of the light flash. Recently Webb[5] has shown that the same kind of result is also obtained with the photographic plate when the frequency of flash is above the fusion frequency. The present work is a continuation of that of Beams and includes the range from 10^{-2} to 8×10^{-9} sec.

[1] Fox Talbot, Phil. Mag. **5**, 21 (1834).
[2] E. P. Hyde, Bul. Nat. Bur. Stand. **2**, 1–32 (1906).
[3] Chariton and Lea, Proc. Roy. Soc. **A122**, 304 (1929).
[4] J. W. Beams, J. O. S. A. **25**, 48 (1934).
[5] J. H. Webb, J. O. S. A. **26**, 347 (1936).

APPARATUS

The apparatus used to produce the very short flashes of light is indicated in Fig. 1. Light from the source, S, which consisted of a straight-filament electric lamp operated from a storage battery passed through the lens, L, was reflected from the rotating mirror, M, and came to focus on the slit, S'. M, made of stellite, was mounted on an air-driven rotor[6] whose speed was varied from 100 to 1500 revolutions per second. These rotational speeds were measured by means of a stroboscopic disk and were very constant for any given pressure of the driving air. Also the rotor was so free of precession and vibration that the image always fell at the same place on the slit.

The duration of each flash was the time taken by the image of the filament to cross the slit or the width of the slit plus the width of the image divided by the velocity of the image, while the number of flashes per second was equal to the r.p.s. of the mirror. The response of the eye to the above light flashes was compared to its response to light from a comparison source which was continuous. For the comparison source light from the same filament S was reflected from a stationary mirror, mounted just above the rotating mirror M through two partially crossed Nicols and to the right-angle prism P (see inset, Fig. 1) situated just in front of the slit S'. In this way it was possible to get into the field of view of the eyepiece two vertical narrow strips of light one just above the other, the lower being intermittent and the upper being continuous.

[6] J. W. Beams, R. S. I. **1**, 667 (1930).

The intensity of the continuous source was adjustable by means of the two Nicols and a photometric "match" could be made so that it was almost impossible to distinguish the boundary between the two strips of light. The comparison beam was carefully tested to make sure that there was no appreciable amount of polarized light in it before it entered the first Nicol prism. The precision of the photometric settings was determined by spinning the mirror M at some fixed speed rapid enough to avoid the flicker effect and setting the upper Nicol until a photometric match was obtained. From the reproducibility of the settings of the Nicol and the law of Malus, it was found that the probable error was less than 1.5 percent. The eye was adapted and rested (in all tests and experiments) and the absolute value of the intensity of the light entering the eye was varied over wider ranges than those used in the regular experiment.

Production of the Flashes

For the production of the shortest flashes it was desirable to keep the slit and the image as small as possible and to have a comparatively long distance between the revolving mirror and the slit. For an image width of 0.018 cm, a slit width of 0.006 cm, a distance from slit to M of 150 cm, and a rotational speed of 1500 r.p.s. for the mirror, the time of flash was 8×10^{-9} sec. In order to produce longer flashes several things were varied: First, the distance from the mirror to the slit; second, the width of the image; and third, the width of the slit. The width of the slit was kept small enough to secure a retinal image in the region of the fovea so that the greatest distinctness of vision could be attained. By varying these three quantities different ranges could be studied. Care was taken to make certain that the longest flash in one range was longer than the shortest flash of the next as the rotational speed of the rotor was varied from its maximum to its minimum speed. As far as possible the ranges studied were produced in all three of the above mentioned ways.

For the production of flashes longer than 10^{-6} sec. it was necessary to use a larger image at S'. To do this a General Electric Mazda strip filament lamp with a filament width of 2 mm was used. Also for the longer flashes it was desirable to reduce the speed of the rotating mirror below that convenient with the air-driven rotor. Consequently a four-sided quartz prism whose faces were optically polished and silvered was mounted directly on the shaft of a variable speed motor. This replaced the mirror mounted on the air turbine. Otherwise there were no essential changes in the apparatus. In this way it was possible to get flashes as long as 3.5×10^{-4} sec.

In order to work in the range from 10^{-4} to 10^{-2} sec., an entirely different arrangement of apparatus was necessary. The light from a straight filament lamp (Fig. 2) was focused by means of a lens on a sectored disk which was mounted on the shaft of the variable speed motor mentioned above. There were 100 openings in the disk, each being about 1.5 mm wide and the distance from the center of the disk to the center of each opening was 13.3 cm. The light passing through the opening in the disk was, in turn, focused on the slit directly in front of the eyepiece. The time of flash in this case was that required for the slit in the disk to pass the image of the filament. In this way the range from about 6×10^{-5} sec. to flashes longer than 10^{-2} sec. was studied. The photometric comparison was made as before.

Procedure and Results

The eye was placed in front of the eyepiece in such a way as to see the narrow slit of light, the lower part of which was from the intermittent, and the upper part from the continuous source. With the eye in this position the rotational speed of the mirror was changed over the maximum range. Although the length of the flash varies

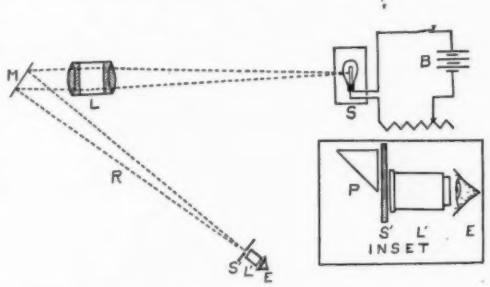

Fig. 1. B, storage battery; L', eyepiece; L, long, focal-length lens; E, observer's eye; S', slit; S, light source; M, rotating mirror; P, prism.

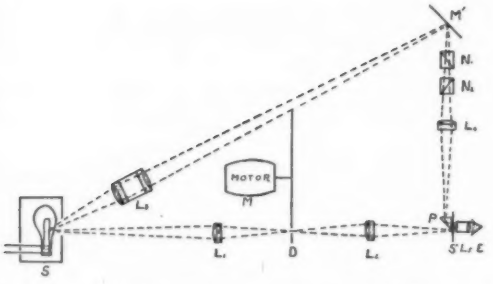

FIG. 2. S, light source; P, right-angled prism; M', mirror; S', slit; E, observer's eye; M, motor; L_1, L_2, L_3, L_4, lenses; L_5, eyepiece; N_1, N_2, Nicols; D, sectored disk.

inversely as this speed, the amount of energy per second received by the eye remains constant. The intensity was observed during this process in the following manner. While the mirror was revolving at a given speed the upper Nicol was turned until a "match" was made between the variable and the continuous light as seen in the field of view of the eyepiece. This was repeated for a number of different speeds, about half of which were as fast as it was safe to run the rotor and the other half as slow as it could be run. In this way the intensities of the light flashes of different lengths were compared with a continuous source.

In all of the tests made over the complete range from 10^{-2} sec. to flashes as short as 8×10^{-9} sec. no change of intensity was observed nor was there any change indicated by the setting of the Nicol which did not fall within the limits to be expected by the probable error of 1.5 percent.

CONCLUSION

From the above results it is evident that, within the limits of error of the experiment, there is no variation in the intensity of light from an intermittent source due to the change in the length of the light flash from 10^{-2} sec. to flashes as short as 8×10^{-9} sec., provided that the eye is rested, is adapted, is receiving the flashes fast enough to avoid the sensation of flicker, and that the intensity is well above that necessary to produce vision. The brightness of the image used varied from approximately 5 to 50 candles/(meter)^2. In other words, the intensity as interpreted by the eye is independent of the time of flash and only dependent on the amount of energy received per second, provided the above conditions are met.

The author wishes to express his appreciation of the help and suggestions of Professor J. W. Beams and other members of the physics staff of the University of Virginia during the course of this work.

The Doppler Effect Considered in Relation to the Michelson-Morley Experiment

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(Received September 9, 1937)

IN an earlier paper[1] I have shown that a null result in the Michelson-Morley experiment can be explained on the assumption of contractions of the apparatus in the ratio

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{n+1} : 1$$

in the direction of motion and

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{n} : 1$$

at right angles to the direction of motion. The results of the Kennedy-Thorndyke experiment are then explained if the clock at the origin is altered in frequency in the ratio

$$[(1-v^2/c^2)^{\frac{1}{2}}]^{1-n} : 1.$$

The value of n in these expressions is not determinable from the Michelson-Morley type of experiment.

I consider in this paper an optical experiment which is competent to decide the value of n. This experiment is the observation of the frequencies of the displaced lines in the Doppler effect, as obtained for instance with canal rays.

We start with the "classical" formula for the Doppler effect, which assumes frequencies to be independent of motion. In its complete form it is

$$\nu' = \nu \frac{1-(v_0/c)\cos\varphi}{1-(v_s/c)\cos\varphi}, \tag{1}$$

where (Fig. 1), ν' is the observed frequency for the observer on particle o, ν is the natural frequency of the light sources; v_s and v_0 are the velocities of source and observer respectively, with respect to the ether, c the velocity of light, and φ the angle between the axis of motion of the (parallel) moving bodies, and the direction of the light ray between them.[2]

We now modify the formula (1) in the manner called for by the results of the Michelson-Morley and Kennedy-Thorndyke experiments. We note that ν becomes, in terms of ν_0, the frequency of the source when stationary in the ether,

$$\nu_s = \nu_0[(1-v_s^2/c^2)^{\frac{1}{2}}]^{1-n}, \tag{2}$$

and that the observed frequency is re-evaluated in terms of the frequency of the observer's standard in the ratio

$$1/[(1-v_0^2/c^2)^{\frac{1}{2}}]^{1-n}, \tag{3}$$

so that we have for our general Doppler effect formula:

$$\nu' = \nu_0 \left[\frac{(1-v_s^2/c^2)^{\frac{1}{2}}}{(1-v_0^2/c^2)^{\frac{1}{2}}}\right]^{1-n} \frac{1-(v_0/c)\cos\varphi}{1-(v_s/c)\cos\varphi}. \tag{4}$$

We shall now study the phenomena to be expected for different values of n in the above formula. This we shall do by taking certain selected values, namely $n=1$, which corresponds to a contraction of the Michelson-Morley apparatus in the ratio $1-(v^2/c^2) : 1$ in the direction of motion; and $n=0$, which corresponds to the contraction of the Michelson-Morley apparatus proposed by Fitzgerald, namely in the ratio $(1-v^2/c^2)^{\frac{1}{2}} : 1$ in the direction of motion.

The case, $n=1$, to be first considered, obviously corresponds to the "classical" formula (1). Taking that formula let us consider first the phenomena to be expected by end-on observation of a moving light source, that is where $\varphi=0$ and II. For the sake of concreteness, we shall imagine we are observing a canal-ray tube, exhibiting in the spectroscope an undisplaced line and a displaced line; v_s is then the absolute velocity of the canal rays, v_0 that of observer and the "stationary" particles; v_s and v_0 being in the same direction.

Putting $\varphi=0$ we get, for the frequency of the

[1] "Graphical Exposition of the Michelson-Morley Experiment," J. Op. Soc. Am. **27**, 177 (1937).
[2] See Born, *Einstein's Theory of Relativity*, p. 109, except for the factor cos φ.

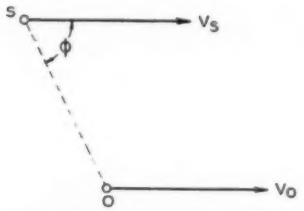

FIG. 1. Representation of parallel moving source (s) and observer (o) for obtaining formula for Doppler effect.

displaced line,

$$\nu' = \nu_0 \frac{1-(v_0/c)}{1-(v_s/c)}$$

$$= \nu_0\left(1-\frac{v_0}{c}\right)\left(1+\frac{v_s}{c}+\frac{v_s^2}{c^2}+\cdots\right) \quad (5)$$

$$= \nu_0\left[1+\frac{(v_s-v_0)}{c}+\frac{v_s}{c}\frac{(v_s-v_0)}{c}+\cdots\right].$$

Putting v_c for the velocity of the canal rays relative to the observer, i.e. v_c is the velocity as calculated from the applied field, we get

$$v_s - v_0 = v_c$$
$$v_s = v_c + v_0,$$

giving for (5)

$$\nu' = \nu_0\left[1+\frac{v_s}{c}+\frac{(v_c+v_0)}{c}\frac{v_c}{c}+\cdots\right]$$

$$= \nu_0\left[1+\frac{v_c}{c}+\frac{(v_c+v_0)}{c}\cdot\frac{v_c}{c}+\cdots\right] \quad (6)$$

$$= \nu_0\left[1+\frac{v_c}{c}+\frac{v_c^2}{c^2}+\frac{v_0 v_c}{c^2}+\cdots\right]$$

and for $\varphi = \Pi$

$$\nu' = \nu_0\left[1-\frac{v_c}{c}+\frac{v_c^2}{c^2}+\frac{v_0 v_c}{c^2}\right]. \quad (7)$$

We therefore have as a first-order effect, displaced lines at + and $-\nu_0(v_c/c)$, that is symmetrical displacements for the two directions of observation; to these are added second-order displacements, which introduce a lack of symmetry which is a function of the velocity of the system as a whole.

Consider next the case of observation at right angles to the direction of motion of the canal rays. Here, due to aberration, our actual angle of observation is $\cos^{-1}(v_0/c)$, which put in formula (1), gives

$$\nu' = \nu_0\left[\frac{1-(v_0^2/c^2)}{1-(v_s v_0/c^2)}\right]$$

$$= \nu_0\left[1-\frac{v_0^2}{c^2}+\frac{v_s v_0}{c^2}\cdots\right]$$

$$= \nu_0\left[1+\frac{v_0}{c}\left(\frac{v_s-v_0}{c}\right)\cdots\right]$$

$$= \nu_0\left[1+\frac{v_0 v_c}{c^2}\cdots\right]. \quad (8)$$

We should therefore expect to observe a displaced line, whose direction of displacement depends on the direction of motion of the system or of the canal rays, as either is changed.

In order to illustrate these conclusions on the Doppler effect when $n=1$ the content of the above equations is exhibited in Fig. 2, for arbitrarily chosen values, $v_c/c=0.1$, $v_0/c=0.2$, and the canal-ray tube turned end for end (changing the sign of v_c). From this figure it is evident that unsymmetrical shifts of the displaced canal ray lines are to be expected, from which the motion of the observing platform with

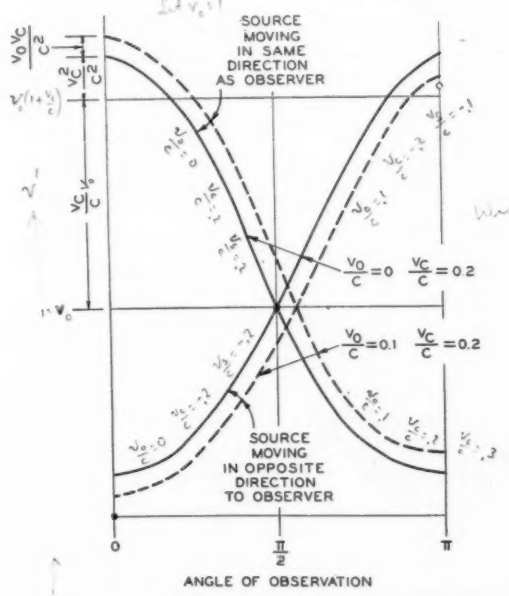

FIG. 2. Frequency of displaced canal-ray line as function of angle of observation and direction of rays, for the case of $n=1$.

respect to the ether could be deduced. The center of gravity of the two displaced lines, obtained by observations at 0° and 180°, will be displaced toward higher frequencies for canal rays moving in the same direction as the system, and to lower frequencies for the opposite direction.

For lateral observation, no displacement is observed when the canal rays are viewed at the aberration angle associated with the velocity of the observer. Part of the unsymmetrical character of the curves in Fig. 2 is due, it will be observed, to the second-order terms in v_c/c, which arise from plotting in terms of *frequencies*. The behaviors characteristic of the motion of the system are more clearly differentiate l if the plot is made in terms of wave-length as is done in Fig. 3. This plot shows that for the case of $v_0/c=0$, the displacements are entirely symmetrical. For finite values of v_0/c the center of gravity of the displaced lines is toward the blue for one orientation of the canal-ray tube, toward the red for the opposite orientation, by the amount $\lambda_0(v_0v_c/c^2)$.

We now proceed to consider the same problem as it is modified by the choice of the value 0 for n. Formula (4) then becomes

$$\nu'=\frac{\nu_0(1-(v_s^2/c^2))^{\frac{1}{2}}}{(1-(v_0^2/c^2))^{\frac{1}{2}}}\cdot\frac{1-(v_0/c)\cos\varphi}{1-(v_s/c)\cos\varphi}. \qquad (9)$$

We will now consider in turn the cases discussed above by the classical treatment:

For $\cos\varphi=1$ we have

$$\nu'=\nu_0\frac{\left[\left(1-\frac{v_s}{c}\right)\left(1+\frac{v_s}{c}\right)\left(1-\frac{v_0}{c}\right)\left(1-\frac{v_0}{c}\right)\right]^{\frac{1}{2}}}{\left[\left(1-\frac{v_0}{c}\right)\left(1+\frac{v_0}{c}\right)\left(1-\frac{v_s}{c}\right)\left(1-\frac{v_s}{c}\right)\right]^{\frac{1}{2}}}$$

$$=\nu_0\frac{\left[1+\frac{(v_s-v_0)}{c(1-(v_0v_s/c^2))}\right]^{\frac{1}{2}}}{\left[1+\frac{(v_0-v_s)}{c(1-(v_0v_s)/c^2))}\right]^{\frac{1}{2}}} \qquad (10)$$

$$=\nu_0\frac{1+\frac{(v_s-v_0)}{c(1-(v_0v_s/c^2))}}{\left[1-\frac{(v_0-v_s)^2}{c^2(1-(v_0v_s/c^2))^2}\right]^{\frac{1}{2}}}.$$

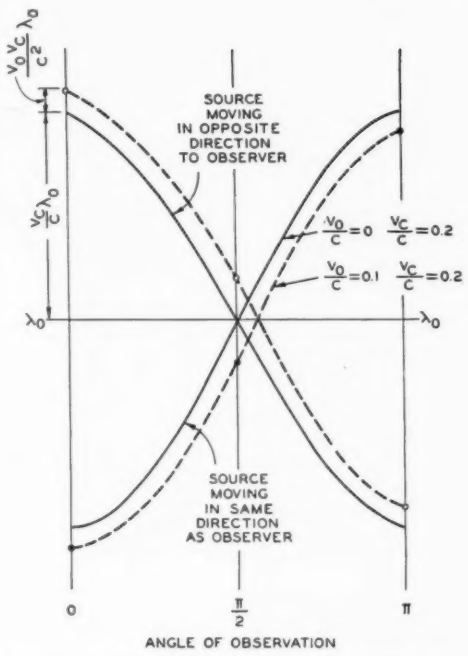

FIG. 3. Wave-length of displaced canal-ray line as function of angle of observation and direction of rays, for the case of $n=1$.

It becomes necessary at this point to evaluate the term

$$\frac{v_s-v_0}{(1-(v_0v_s/c^2))}. \qquad (11)$$

From a previous paper[3] we take the result that in a system where clock rates vary in the manner now under consideration, this expression is a function of the *observed* difference of velocities, which becomes identical with this observed difference, when the clocks used in observing the passage of one body past the other are moved infinitely slowly; (11) is thus for the latter case simply the observed velocity of the canal rays, as obtained for instance by the measurement of field strength in the tube. We designate this observed velocity by the symbol used in the classical case, v_c. We therefore have

$$\nu'=\nu_0\frac{1+(v_c/c)}{[1-(v_c^2/c^2)]^{\frac{1}{2}}}. \qquad (12)$$

[3] "Light Signals on Moving Bodies as Measured by Transported Rods and Clocks," J. Op. Soc. Am. **27**, 263 (1937).

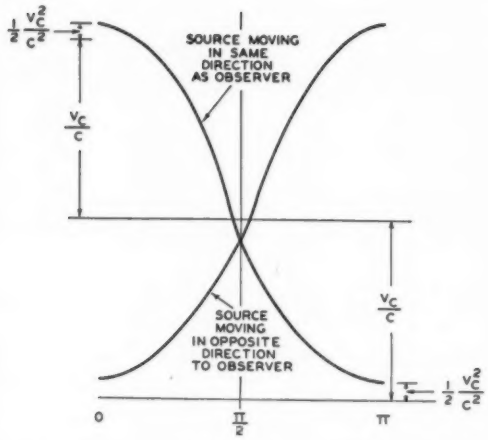

FIG. 4. Frequency of displaced canal-ray line as function of angle of observation and direction of rays for the case of $n=0$

We obtain similarly for $\cos\varphi=-1$

$$\nu'=\nu_0\frac{1-(v_c/c)}{[1-(v_c^2/c^2)]^{\frac{1}{2}}}. \qquad (13)$$

Let us next turn our attention to observation perpendicular to the direction of motion of the bodies. For this case, taking into account aberration, which makes the actual angle of observation $\cos^{-1}(v_0/c)$, we have

$$\nu'=\nu_0\frac{(1-(v_s^2/c^2))^{\frac{1}{2}}}{(1-(v_0^2/c^2))^{\frac{1}{2}}}\cdot\frac{1-(v_0^2/c^2)}{1-(v_0v_s/c^2)}$$

$$=\nu_0\left(1-\frac{(v_s-v_0)^2}{c^2(1-(v_0v_s/c^2)^2}\right)^{\frac{1}{2}} \qquad (14)$$

$$=\nu_0\left(1-\frac{v_c^2}{c^2}\right)^{\frac{1}{2}}.$$

The formula indicates that for lateral observation at apparent 90° from the direction of the discharge the frequency is *reduced*.

Summarizing, we find that the frequencies of the canal ray lines are symmetrically displaced, except for the second-order term $(1-(v_c^2/c^2))^{\frac{1}{2}}$, which acts to *increase* the frequency as given by the first-order terms for "end on" observation in either direction, and to *decrease* it for lateral observation. These relations are exhibited in Fig. 4. Compared with the case for $n=1$, the changes in frequency are seen to be simply a *decrease* for all conditions of observation by the factor $\frac{1}{2}(v_c^2/c^2)$, from what would occur if, with $n=1$, the observer were stationary in the ether. When expressed in terms of wave-length instead of frequency, as was done for the case of $n=1$ in Fig. 3, these relations reduce to a simple increase of wave-length for all conditions of observation in the ratio $1/(1-(v^2/c^2))^{\frac{1}{2}}:1$, over the stationary classical case.

Certain important general conclusions are embodied in formulae (13) and (14). We note that the phenomena are described entirely in terms of *observed relative motions* of light source and observer. Velocities with respect to the ether have dropped out and the phenomena are thus invariant with motion of the system through the ether. (It is evident, without formal proof, that this is the case only for the value 0 for the exponent n.) In this invariance the Doppler effect phenomena thus become similar to the Michelson-Morley phenomena. There is however a vital difference in the character of this invariance. In the case of the Michelson-Morley experiment the function of the term $[(1-(v^2/c^2))^{\frac{1}{2}}]^{n+1}$ is to make the effect not only invariant but *null*, that is, the predicted behavior corresponds to the "classical" case for the apparatus stationary in the ether. In the Doppler effect the result predicted is invariant, but *different* from the classical case for the observer stationary in the ether, as shown by comparing Eqs. (12), (13) and (14), with Eqs. (6), (7) and (8), with v_0 put equal to zero. Experiments on canal rays, of the sort here assumed, should therefore be capable of furnishing positive optical evidence for the validity of the contractions postulated by Fitzgerald, Larmor and Lorentz.[4]

[4] The crucial character of such experiments for deciding between the several possibilities for explaining the Michelson-Morley experiment (entrained ether, ballistic character of light emission, contractions of matter in traversing the universal pattern of radiant energy) was pointed out by Einstein and by Ritz thirty years ago.

Note on the Computation of Optical Constants

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IN a series of admirable papers on "Plane Waves of Light," Fry[1] has deduced an expression enabling one to compute the optical constants of a metal, namely,

$$N+iK_0=\tan I\left[\frac{(\rho+1)^2-4\rho\sin^2 I}{(\rho+1)^2}\right]^{\frac{1}{2}}. \quad (1)$$

The above is Fry's Eq. (55), but in the corrected form. In this equation N is the refractive index of the metal; K_0 is the extinction coefficient; $\rho=e^{i\phi}\tan\theta$, where ϕ is the relative phase difference of the components of the light vector vibrating perpendicular and parallel to the plane of incidence, and θ is the angle of restored plane polarization; I is the angle of incidence of the light on the metal surface.

Equation (1) has been employed recently by Ives and Briggs[2] to evaluate the optical constants of potassium and sodium in the visible and ultraviolet regions of the spectrum. The equation has also been used by Emberson[3] in the study of the optical constants of mercury, and in the computation of film thicknesses of sputtered silver and platinum. Since the writer has found the numerical results as computed with the aid of Fry's equation to come out identical with those computed by means of Drude's equations,[4] it is thought worth while to show that the equations are identical in form.

Taking the complex conjugate of Fry's Eq. (1),

$$N-iK_0=\tan I\left[1-\frac{4\rho'\sin^2 I}{(\rho'+1)^2}\right]^{\frac{1}{2}}, \quad (2)$$

where ρ' is the complex conjugate of ρ. In order to conform with the Drude notation, let us change I to ϕ; ϕ to Δ; θ to $90-\psi$, where ψ is the azimuth of restored plane polarization, whose value is given by Drude's relation, $\tan\psi=\rho$. The ρ now used must not be confused with that previously defined by Fry. Also N should be changed to n, and K_0 to nk. With these changes of notation in mind, Fry's Eq. (2) can be transformed into Eq. (3), expressed in Drude's notation.

$$n(1-ik)=\tan\phi\left[1-\frac{4\rho e^{i\Delta}\sin^2\phi}{(1+\rho e^{i\Delta})^2}\right]^{\frac{1}{2}}. \quad (3)$$

From Drude's *Theory of Optics* (English translation), page 362, Eq. (15), we have,

$$\frac{1+\rho e^{i\Delta}}{1-\rho e^{i\Delta}}=\frac{\sin\phi\tan\phi}{(\epsilon'-\sin^2\phi)^{\frac{1}{2}}}, \quad (4)$$

where ϵ' is defined as the complex dielectric constant whose value is given by $\epsilon'=n^2(1-ik)^2$. Squaring (4), transposing terms, and solving for $n(1-ik)$, we have,

$$n(1-ik)=\sin\phi\left[1+\tan^2\phi\frac{(1-\rho e^{i\Delta})^2}{(1+\rho e^{i\Delta})^2}\right]^{\frac{1}{2}}$$

$$=\tan\phi\left[1-\frac{4\rho e^{i\Delta}\sin^2\phi}{(1+\rho e^{i\Delta})^2}\right]^{\frac{1}{2}}, \quad (5)$$

which is identical with Eq. (3).

The writer has computed the optical constants of sodium and potassium for $\lambda=5461$A, from the data given by Ives and Briggs,[2] using the form of equations given by Drude,[4] suitable for logarithmic computation. Letting

$$n^2(1-ik)^2-\sin^2\phi=S^2e^{-2iQ},$$

it follows from Eq. (4) on equating real and imaginary quantities, that $\tan Q=\sin\Delta\tan 2\psi$, and $S=\sin\phi\tan\phi\tan P$, where the quantity P is given by the relation $\cos 2P=\cos\Delta\sin 2\psi$. Letting $\tan X=S^2\sin 2Q/(S^2\cos 2Q+\sin^2\phi)$, it follows that

$$k=\tan X/2 \quad \text{and} \quad n=\left[\frac{S^2\cos 2Q+\sin^2\phi}{1-k^2}\right]^{\frac{1}{2}}. \quad (6)$$

For ease and rapidity in logarithmic computa-

[1] Fry, J. O. S. A. **15**, 137 (1928); **16**, 1 (1928); **22**, 307 (1932).
[2] Ives and Briggs, J. O. S. A. **26**, 238 (1936); **27**, 181 (1937).
[3] Emberson, J. O. S. A. **26**, 443 (1936).
[4] Drude, Ann. d. Physik **64**, 159 (1898).

tion, the writer has prepared a number of mimeographed copies of the outline to be followed in the computations. It becomes then a relatively simple matter to fill in the form with logarithms and anti-logarithms, ultimately leading to the computed values of n and k.

If the computations are carried out using Fry's equation, then according to his statement,[5] "it is only necessary to substitute the measured values" (in the defining equation for ρ) "to obtain ρ, and then substitute this ρ" into Eq. (1). "The result will be a complex quantity of which the real part is the index of refraction and the imaginary part is the extinction coefficient." In reality $(\rho+1)^2$ as well as ρ are complex quantities. These must be combined so that Eq. (1) ultimately assumes the form:

$$N+iK_0=\tan I(a+ib)^{\frac{1}{2}}$$
$$=(\tan I)r^{\frac{1}{2}}(\cos \alpha/2+i \sin \alpha/2), \quad (7)$$

where r is the modulus of the complex quantity, and the value of α is to be obtained in the usual manner from a, b, and r. The values of N and K_0 follow after equating real and imaginary quantities. It seems to the writer that this type of computation is more awkward and lengthier than the logarithmic method of Drude, and hence he is inclined to favor the latter's method of computation of optical constants. For potassium, Drude's equations yield values of 0.0911 and 1.423 for N and K_0, respectively, while the results given by Ives and Briggs (using the Fry equation) are 0.091 and 1.42. In the case of sodium, the Drude equations yield values of 0.0294 and 2.316 for N and K_0, respectively, while Ives and Briggs give 0.029 and 2.32. The author has checked Ives and Briggs' computations for sodium using the Fry equation, and has obtained values of 0.0293 and 2.316 for N and K_0, respectively. Computations have been carried out purposely to another decimal place in order to reveal the extent of agreement.

Fry's equation, though apparently different in form from Drude's, is really identical with the latter. This note is not intended as any criticism of Fry's work, but simply to point out to investigators in the field of metal optics, that having once obtained experimental data, it becomes merely a personal choice as to the form of equations to be chosen for computation. The results will come out identical.

[5] Fry, J. O. S. A. **16**, 23 (1928).

Optical Constants of Rubidium and Cesium

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IN previous papers[1] we have presented the results of measurements of the optical constants of potassium and sodium in the ultraviolet and visible ranges, with a description of the apparatus used and the method of making measurements. The present paper deals with the results of measurements of the optical constants of rubidium and cesium by the same methods and for the same wave-length range. As with sodium and potassium, previous investigations of the optical constants of these metals were confined to the visible region of the spectrum. The work of Wood[2] showed the existence of wide transparency bands in the ultraviolet and enabled him to determine the refractive indices for limited ranges. Our original interest lay in the application of these constants to the theory of photoelectric emission from thin films,[3] for which a knowledge of both refractive index and extinction coefficient is necessary.

Two methods for preparing the alkali metal mirrors have been described. In the first method, used with potassium, the metal was in the form of a thick layer between two plates, one of fused quartz, the other of glass. Measurements were made on the quartz potassium surface through a 60° fused quartz prism sealed to the mirror assembly with paraffin oil. The second method, used with sodium, employed a free surface of the metal formed on the flattened surface of a bulb enclosed in an evacuated tube. The incident and emergent beams were accommodated by quartz windows. The rubidium mirrors used in this work were of the "sandwich" type, similar to those used with potassium; the cesium mirrors were of the free surface type. Details concerning the construction of the tubes and the technique of making the mirrors are given in the papers to which reference has been made. The metallic rubidium and cesium were prepared by the reduction of their chlorides with calcium in a separate vacuum apparatus. After distillation the metal was collected in several thin walled glass tubes, ready for introduction into the final distilling train which preceded the main tube in which the mirror was formed.

RUBIDIUM

Qualitative spectrographic analysis of the rubidium chloride used showed the presence of a small amount of potassium, with traces of sodium, cesium, calcium, silicon and aluminum. Similar analysis of the metallic rubidium after two distillations showed only potassium and a trace of sodium. Four stages of distillation were used in introducing the metal into the tube where the mirror was formed, the temperature at each distillation stage being kept as low as possible.

Two rubidium mirrors were made, both of which appeared very good visually over their entire surface. The first mirror lasted but two days, signs of deterioration appearing the second day after being taken from the vacuum. A transparent spot appeared near one edge, apparently due to a leak through the paraffin wax, and gradually spread to the area utilized for reflection. The second mirror lasted five days before going bad in a similar way.

The accompanying measurements, Tables I and II, were all obtained from the second mirror; those for λ5780, 5461 and 4358 being obtained visually, the ones for λ4047, 3650, 3341, 3126, 3022 and 2536 being obtained photographically.

Figure 1 shows our values of N and K_0 for rubidium, together with several values in the visible range obtained by Nathanson.[4] Some values of N calculated from the curve given by Wood for the reflectivity of rubidium are also given. Our curves for rubidium show the same general characteristics as those we gave for potassium; i.e., a low value for the refractive index in the visible is followed by a sharply increasing value as we proceed into the ultraviolet,

[1] Herbert E. Ives and H. B. Briggs, J. O. S. A. **26**, 238 (1936), and also J. O. S. A. **27**, 181 (1937).
[2] R. W. Wood, Phys. Rev. **44**, 353 (1933).
[3] Herbert E. Ives, Phys. Rev. **38**, 1209 (1931).
[4] J. B. Nathanson, Phys. Rev. **11**, 227 (1918).

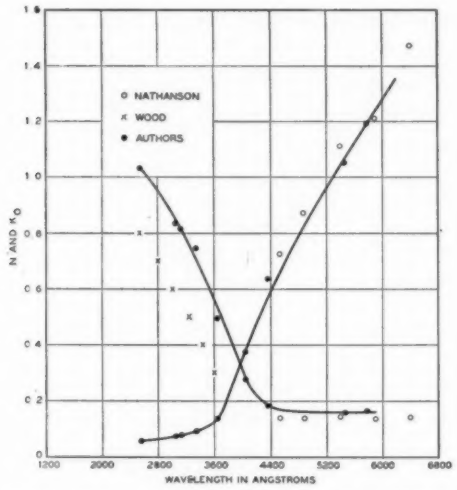

FIG. 1. Dispersion curves of N and K_0 for rubidium.

whereas the extinction coefficient decreases rapidly from large values in the visible to very small values in the ultraviolet. The intersection of the N and K_0 curves is very near λ4000.

Figure 2 shows the reflecting power of rubidium for normal incidence, calculated from the formula

$$R=\frac{(N-1)^2+K_0^2}{(N+1)^2+K_0^2}.$$

The curve is very similar to that obtained for potassium, with the exception that it is shifted about 700A toward the red.

Figure 3 shows the principal incidence angle $\bar{I}$, and the principal azimuth $\bar{\theta}$. The principal incidence shows a distinct minimum at λ3600. The principal azimuth is fairly constant at values a few degrees above 45° in the visible. It increases sharply toward 90° in the near ultraviolet. The principal azimuth appears to change most rapidly while the principal incidence is going through its minimum.

TABLE I. *Experimental data for rubidium.*

λ	I	Φ	θ
5780	60°	76° 48′	46° 48′
5461	60	73° 36′	46° 41′
4358	60	65° 24′	46° 30′
4047	60	61° 6′	46° 31′
3650	60	56° 25′	46° 8′
3341	60	50° 35′	46° 21′
3126	60	48° 25′	46° 21′
3022	60	47° 44′	46° 18′
2536	60	39° 10′	46° 37′

In Table III and Fig. 4 are shown the results of calculations of the transmission of several thicknesses of rubidium film. The calculations were made using the formula for reflection and transmission developed by Dr. Fry.[5] For the purposes of this calculation the rubidium film is assumed to be in air. We note the very decided increase in transmission in the near ultraviolet, which is in accord with the photographic results of R. W. Wood. In this connection we wish to call attention to the fact that the transmission curves for rubidium, as well as those for potassium and cesium, give little justification for such expressions as "the point where transparency commences" when referring to the transmission of light by the alkali metals. The implication that the beginning of transparency is abrupt is not borne out by experiment.[6]

CESIUM

Qualitative spectrographic analysis of the cesium chloride used showed impurities of sodium and calcium, with traces of strontium, barium, and magnesium. As with rubidium, four stages of distillation were used in introducing the metal into the tube, the temperature during each distillation being kept as low as possible.

The type of mirror used in the work on cesium was similar to that used with sodium, i.e., a free

TABLE II. *Values calculated from experimental data for rubidium.*

λ	N	K_0	$R(\%)$	$\bar{I}$	$\bar{\theta}$
5780	0.164	1.19	76.4	59° 38′	47° 36′
5461	0.157	1.05	74.2	57° 55′	47° 36′
4358	0.181	0.636	59.7	52° 1′	48° 16′
4047	0.275	0.373	37.6	47° 22′	49° 41′
3650	0.496	0.135	12.1	35° 37′	57° 33′
3341	0.745	0.090	2.39	37° 35.5′	80° 36′
3126	0.814	0.078	1.23	39° 39′	83° 17′
3022	0.833	0.071	0.98	40° 11.5′	84° 6′
2536	1.031	0.056	0.99	46° 0′	86° 48′

[5] Thornton C. Fry, J. O. S. A. **16**, 1 (1928).

[6] In our previous reports on the optical constants of potassium and of sodium, comparisons were made between the constants as derived from our experimental work, and those calculated from the theory of metallic dispersion as developed by Kronig (R. deL. Kronig, Proc. Roy. Soc. **A133**, 255 (1931)). Agreement between experimental and theoretical values, which was good for sodium and fair for potassium, was poor for rubidium and worse for cesium.

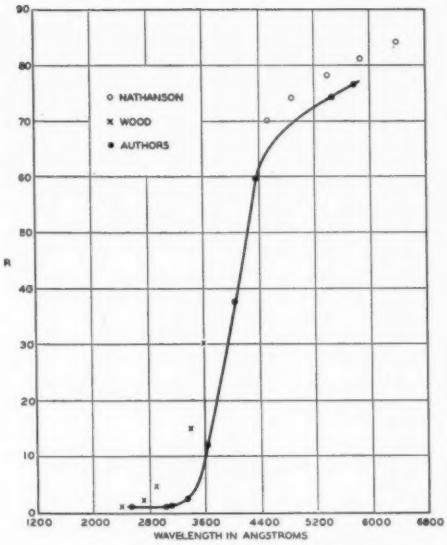

FIG. 2. Reflecting power of rubidium.

surface of the metal deposited on the flattened surface of a bulb enclosed in an evacuated tube. Details of the preparation of the surface, with minor variations due to the lower melting point of the cesium, are similar to those described for sodium. As it seemed desirable to preserve the sodium surface, a new tube was made for the cesium differing only from the sodium tube previously described in the improved quality of the windows and the method of attaching them to the tube. There were available two circular fused quartz windows, $1\frac{7}{8}''$ in diameter and $\frac{1}{8}''$ thick of exceptional quality with regard to freedom from strains. Also available were two quartz step seal to Pyrex tubes, $1\frac{1}{4}''$ in diameter. The quartz windows were attached to the tubes

TABLE III. *Transmission of rubidium.*

	THICKNESS OF FILM (CM)			
λ	1.25×10^{-5}	2.5×10^{-5}	5.0×10^{-5}	7.5×10^{-5}
5780	0.112	4.6×10^{-3}	7.19×10^{-6}	1.11×10^{-8}
5461	.140	7.4×10^{-3}	1.80×10^{-5}	4.40×10^{-8}
4358	.215	1.43×10^{-2}	2.25×10^{-4}	2.30×10^{-6}
4047	.305	6.30×10^{-2}	3.35×10^{-3}	1.84×10^{-4}
3650	.472	2.45×10^{-1}	8.13×10^{-2}	2.47×10^{-2}
3341	.615	4.22×10^{-1}	1.78×10^{-1}	7.55×10^{-2}
3126	.655	4.54×10^{-1}	2.05×10^{-1}	9.49×10^{-2}
3022	.674	4.71×10^{-1}	2.22×10^{-1}	1.08×10^{-1}
2536	.708	5.02×10^{-1}	2.51×10^{-1}	1.26×10^{-1}

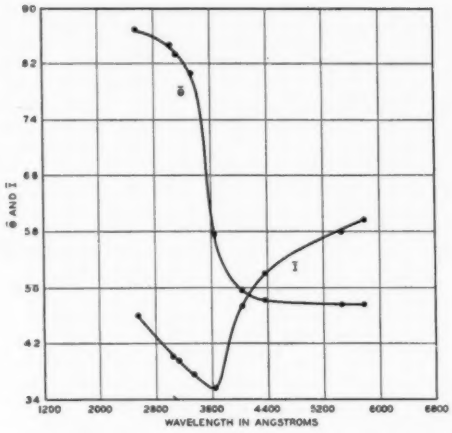

FIG. 3. Principal incidence angle and principal azimuth for rubidium.

with silver chloride, following substantially the method described by Hulburt.[7]

The quartz ends of the step seal tubes were first ground flat to receive the windows. Silver chloride was then placed on the ground surface and heated to melting, after which it was spread evenly over the surface with a glass rod. The window was then placed and heat applied directly at the seal until the silver chloride was partly reduced, showing a brownish color. Complete reduction of the silver chloride was avoided as it invites strains and may result in cracking of window or tube. These seals were quite satisfactory from the vacuum standpoint, and withstood baking at a temperature slightly over 400°C.

Before sealing the windows to the tube they were examined over their whole area between crossed Nicols. With the exception of the effect due to the minute air bubbles which occur in all fused quartz the field showed scarcely any perceptible brightening due to the quartz. After making the silver chloride seals the windows were again tested for strain. A mirror of evaporated gold was supported on the specimen table in the position to be occupied by the cesium surface, and the tube with sealed in windows was lowered over the gold mirror. Spot patterns for the light passing through the two windows and reflected from the gold were obtained for wave-

[7] E. O. Hulburt, Science **54**, 147 (1922).

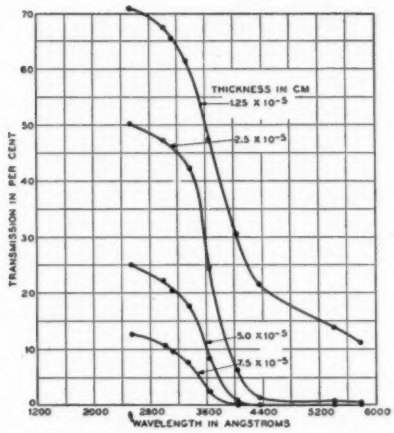

FIG. 4. Calculated transmissions for several thicknesses of rubidium.

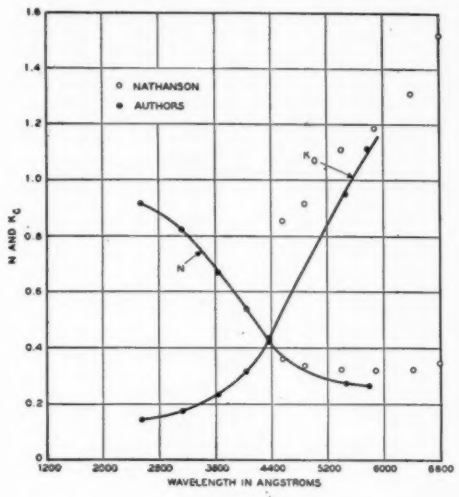

FIG. 5. Dispersion curves of N and K_0 for cesium.

lengths λ3650 and λ3126. The tube was then removed without disturbing the gold mirror and the spot pattern again photographed. Comparison of phase and azimuth for these two sets of patterns showed no changes greater than measurement errors (±1° for single spots).

In order to determine if measurable strains were introduced by evacuation similar tests were made. The completed tube, previous to the introduction of the cesium, was pumped out, adjusted on the specimen table, and spot patterns obtained by reflection from the flattened surface of the bulb. A point flame was then used to puncture the seal-off tube allowing air to enter the tube without altering its adjustment, after which spot patterns were again obtained. Comparison of the coordinates of corresponding spots for the two sets of plates showed that the central portions of the windows were not strained. Near the seals strains existed which gave shifts up to 2½°. As the aperture of the windows was large compared to that of the beam, and the strains were confined to a region near the seals, it was only necessary to center the beam with respect to the windows to avoid the edge strains.

No difficulty was experienced in producing specular surfaces of cesium of very good quality, but some care was necessary to preserve these surfaces for a period of days. During the preparation of the surface, the bulb upon which the metal was deposited was kept a few degrees below room temperature by blowing cold air through it. Light flaming of the walls of the tube caused the cesium to deposit rapidly on the bulb, the operation being continued until a film of sufficient thickness had been attained. The new surface was highly specular and showed a distinctly golden luster. On being allowed to attain room temperature the surface lost its specular character to a large degree, becoming more and

TABLE IV. *Experimental data for cesium.*

λ	I	Φ	θ
5780	60°	87° 1′	49° 14′
5461	60	81° 14′	49° 35′
4358	60	62° 26′	50° 48′
4047	60	55° 47′	51° 36′
3650	60	47° 21′	52° 46′
3126	60	34° 37′	56° 6′
2536	60	25° 43′	59° 39′

TABLE V. *Values calculated from experimental data for cesium.*

λ	N	K_0	$R(\%)$	$\bar{I}$	$\bar{\theta}$
5780	.264	1.123	63.1	58° 54′	49° 17′
5461	.278	.950	56.1	56° 39′	49° 47′
4358	.425	.438	23.5	47° 53′	53° 24′
4047	.540	.320	12.7	44° 4′	57° 25′
3650	.671	.233	5.72	40° 41′	66° 9′
3126	.827	.174	1.79	41° 49′	76° 29′
2536	.916	.143	0.74	43° 36′	80° 39′

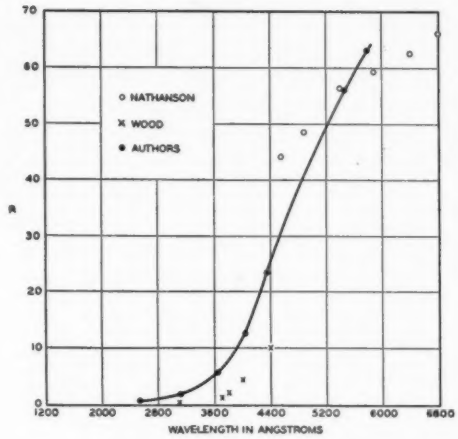

FIG. 6. Reflecting power of cesium.

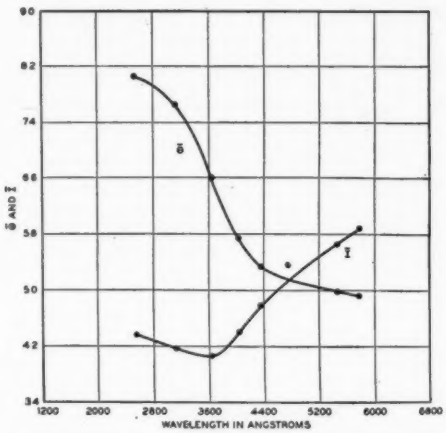

FIG. 7. Principal incidence angle and principal azimuth for cesium.

more matt. Surfaces could be preserved for several days by packing the inner flattened bulb with CO_2 snow immediately after the surface was formed and renewing the snow from time to time. Even so there was a slight tendency to go matt with time, probably due to condensation of additional cesium on the mirror surface.

In order to determine if there was any measurable difference between the phase shift Φ and the azimuth θ for a partly matt surface as compared with a specular one, negatives at λ4047 and 3650 were obtained using a surface that had become decidedly matt by remaining over the week-end without carbon dioxide snow. For the matt surface the phase shift was slightly less than for the specular, the difference being less than one degree. No change in azimuth was detected.

Table IV gives the experimental results. Negatives for wave-lengths 2536 to 4047 were obtained with the apparatus set up as previously described. A small modification in the Babinet made it possible to extend the photographic method to include the visual range. With the exception of λ2536, eight plates were taken at each wave-length, four comparison plates and four reflection plates. At λ2536, where very long exposure time was necessary, only two plates were taken, one reflection plate and one comparison plate.

Our values of N and K_0 for cesium are shown in Fig. 5, together with values in the visual by Nathanson.[8]

The reflection curve, Fig. 6, shows a pronounced drop in reflecting power occurring in the visible range. Comparison with the other alkali

TABLE VI. *Transmission of cesium.*

λ	Thickness of Film (cm)		
	1.25×10^{-5}	2.5×10^{-5}	5.0×10^{-5}
5780	.117	5.74×10^{-2}	1.30×10^{-2}
5461	.151	1.03×10^{-2}	4.38×10^{-2}
4358	.273	5.19×10^{-2}	4.04×10^{-3}
4047	.318	8.06×10^{-2}	7.21×10^{-3}
3650	.369	1.33×10^{-1}	1.80×10^{-2}
3126	.416	1.76×10^{-1}	3.03×10^{-2}
2536	.414	1.72×10^{-1}	2.95×10^{-2}

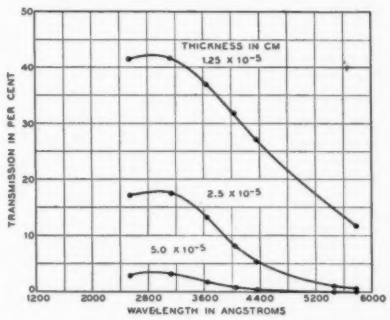

FIG. 8. Calculated transmissions for several thicknesses of cesium.

[8] J. B. Nathanson, Phys. Rev. **25**, 75 (1925).

metals at λ4047 gives the following wide variation in reflecting powers; Cs, 12.7 percent, Rb, 37.6 percent, K, 73.2 percent, Na, 94.5 percent.

In Fig. 7 the principal incidence angle shows a weak minimum at λ3650. The principal azimuth rises less sharply than with the other alkali metals. In the visible range, the angle of principal incidence is highest for sodium and lowest for cesium; for the same range the principal azimuth is lowest for sodium and highest for cesium.

Table VI and Fig. 8 show the transmission of cesium for three thicknesses of film. The increase in transmission with decreasing wave-length is not as sharp as for rubidium or potassium. The curves show a suggestion of a maximum at λ3000.

LETTERS TO THE EDITOR

Illumination Characteristics of the Camera Lens

The article by Mr. Frank Benford in the August issue of this Journal,[1] which has just reached us, contains a personal reference to both of us which calls for a reply. Mr. Benford observes that we "seemingly assumed that all parts of the film would be equally exposed when photographing equally bright objects in various parts of the field." He then demonstrates the $\cos^4\theta$ law and the effect of vignetting, both of which are, as he says, fairly obvious.

Mr. Benford has been perhaps misled by the fact that the three articles which mention the method of photographic photometry which we have adopted do not go into elementary details, as each article is primarily concerned with one particular phase of the subject. It was not mentioned for example that the geometry of the problem permits the use of long focus lenses, so that with our camera the maximum error due to the $\cos^4\theta$ law is about 10 percent at the end of the plate. No important measurements are ever made near the edges of the plate, so that the actual error is smaller—about 5 percent. Furthermore lenses were selected for this work which enables us to use a reasonable aperture without vignetting from the lens mounts. A standard aperture of $f:11$ is used for photometric work, using a Cooke $f:6.8$ anastigmat, or a Ross Xpres $f:4.5$ anastigmat. It is possible to increase the aperture to about $f:9$ or $f:6$ respectively without interference from the mounts with these lenses, but we use $f:11$ to give a good margin of safety. We also measured the transmission of the lens at the relevant angles and confirmed the accuracy of the method by over-all checks.

There are many pitfalls in photographic photometry of which Mr. Benford is no doubt aware; but it is perhaps due to us to record that we foresaw this particular one.

J. M. Waldram
A. Bloch

Research Laboratories of the
General Electric Company, Ltd.,
Wembly, England,
August 21, 1937.

[1] J. O. S. A. **27**, 286 (1937).

Messrs. Waldram and Bloch point out that they foresaw errors of from five to ten percent in their photographic photometry due to the geometric causes discussed in my recent note. I quite agree that under the conditions this size of correction would not be worth making, but their previous investigation of this phase of the work deserved notice in their reports, and a brief mention would have added materially to an otherwise excellent description.

Frank Benford

Research Laboratory,
General Electric Company,
Schenectady, N. Y.,
September 3, 1937.